NAME (print): ____

You are responsible for following the instructions below.

1.) Mark your Math 151 section below.

☐ 501 (TR 5:30) ☐ 502 (TR 3:55) ☐ 503 (TR 2:20)

2.) Print your name, section, and exam form (F.1 or F.2) on the scantron.

3.) You must show your work to receive credit in Part 2.

4.) No calculators.

5.) Turn your cell phone off.

M.C.	16	17	18	19	20	**TOTAL**
45 pts	*8 pts*	*8 pts*	*15 pts*	*16 pts*	*8 pts*	*100 pts*

The volume and surface area of a sphere of radius r are

$$V = \tfrac{4}{3}\pi r^3 \qquad \text{and} \qquad S = 4\pi r^2\,.$$

Part 1

Mark your answer on the scantron. (Since your scantrons are not returned, we suggest that you also circle the answer below.) Each question is worth *3 points*.

1.) At what x-value(s) on the graph $y = 25\arctan x$ is the tangent line parallel to the line $0 = 5x - y + 14$?

- **(a)** $x = 5$
- **(b)** $x = \pm 3$
- **(c)** $x = 0$
- **(d)** $x = \pm 2$
- **(e)** $x = \pm 5$

2.) The derivative of $y = \dfrac{\ln\theta}{\tan\theta}$ is

- **(a)** $y' = \dfrac{1/\theta}{\sec^2\theta}$
- **(b)** $y' = \dfrac{\frac{1}{\theta}\tan\theta + \ln\theta\sec\theta}{\tan^2\theta}$
- **(c)** $y' = \dfrac{\frac{1}{\theta}\tan\theta - \ln\theta\sec^2\theta}{\tan^2\theta}$
- **(d)** $y' = \dfrac{\frac{1}{\theta}\tan\theta - \ln\theta\sec\theta}{\sec^2\theta}$
- **(e)** $y' = \dfrac{\ln\theta\sec\theta - \frac{1}{\theta}\tan^2\theta}{\sec^2\theta}$

3.) The equation of the line tangent to the curve $(1 + 2y)\sec x = (1 + x)e^y$ at the point $(0, 0)$ is

- **(a)** $y = x$
- **(b)** $y = 1$
- **(c)** $y = -x$
- **(d)** $y = x + 1$
- **(e)** $y = -2x$

4.) The position of a particle at time t is $\mathbf{r}(t) = \langle \sqrt[3]{t}, e^{2t} \rangle$. The velocity is

(a) $\langle \frac{1}{3}t^{2/3}, 2e^{2t} \rangle$

(b) $\langle \frac{1}{3}t^{-2/3}, 2e^{2t} \rangle$

(c) $\langle 3t^{-2/3}, 2e^{t} \rangle$

(d) $\langle \frac{1}{3}t^{-2/3}, e^{2t} \rangle$

(e) $\langle \frac{1}{3}t^{-2/3}, 2e^{t} \rangle$

5.) This is a continuation of Question 4. The line tangent to the parametric curve $\mathbf{r}(t)$ at $(1, e^2)$ is

(a) $y = 6e^2 x$

(b) $y = 6x - 5e^2$

(c) $y = 6x - 5$

(d) $y = 6e^2 x - 5$

(e) $y = 6e^2 x - 5e^2$

6.) This is a continuation of Question 4. The acceleration of the particle is

(a) $\langle \frac{2}{9}t^{-1/3}, 4e^{2t} \rangle$

(b) $\langle -\frac{2}{9}t^{-5/3}, 4e^{2t} \rangle$

(c) $\langle -2t^{-5/3}, 2e^{t} \rangle$

(d) $\langle -\frac{2}{9}t^{-5/3}, 2e^{2t} \rangle$

(e) $\langle -\frac{2}{9}t^{-5/3}, 2t^{t} \rangle$

7.) Using the method of linear approximate to estimate $\sqrt{15}$ we find

(a) $\sqrt{15} \approx 9/2$

(b) $\sqrt{15} \approx 15/4$

(c) $\sqrt{15} \approx 31/8$

(d) $\sqrt{15} \approx 29/8$

(e) $\sqrt{15} \approx 4$

8.) Let $g(x)$ be the inverse function of $f(x) = x^7 - 2x^3 - 14x + 5$. Then $g'(5)$ is

(a) 5

(b) -14

(c) $1/14$

(d) $-1/14$

(e) $-1/5$

9.) The derivative of $y = x^{1/x}$ is

(a) $x^{1/x}\dfrac{1}{x^2}(1 - \ln x)$

(b) $x^{1/x}\dfrac{1}{x^2}(x^3 - \ln x)$

(c) $\dfrac{1}{x^2}(1 - \ln x)$

(d) $x^{1/x}\dfrac{\ln x}{x^2}$

(e) $\dfrac{x^{(1-x)/x}}{x}$

10.) The derivative of $y = \arctan(x^2+5)$ is

(a) $\dfrac{2x}{1+(x^2+5)^2}$

(b) $\dfrac{2x}{1+4x^2}$

(c) $\dfrac{5x}{1+(x^2+5)^2}$

(d) $\dfrac{2x}{1+(x^2+5)}$

(e) $\dfrac{2x}{1+2x}$

11.) We may bound the integral $I = \int_0^2 \sqrt{x^3+1}\,\mathrm{d}x$ as follows

(a) $2 \leq I \leq 3$

(b) $1 \leq I \leq 6$

(c) $1 \leq I \leq 3$

(d) $0 \leq I \leq 6$

(e) $2 \leq I \leq 6$

12.) If $\displaystyle\int_{-3}^{1} 2\,f(t)\mathrm{d}t = 6$ and $\displaystyle\int_{4}^{-3} f(t)\,\mathrm{d}t = 10$, then what is $\displaystyle\int_{1}^{4} f(t)\,\mathrm{d}t$?

(a) -16

(b) 4

(c) -13

(d) 7

(e) -4

13.) Below is a graph of the **derivative** $\boldsymbol{y = f'(x)}$. Use the graph to identify the location of the local minimum(s) of the **function** $\boldsymbol{f(x)}$,

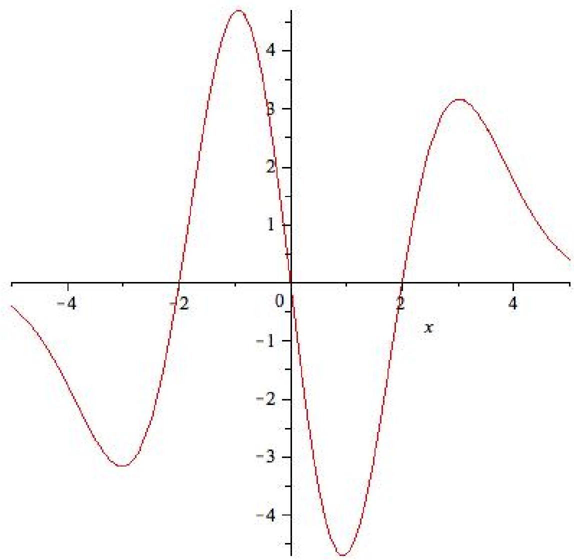
(a) $x = 0$

(b) $x = -3, 1$

(c) $x = \pm 2$

(d) $x = 2$

(e) $x = 1$



14.) Compute $\displaystyle\lim_{t \to \pi/2} \frac{\cos t}{t - \frac{\pi}{2}}$.

(a) Does not exist.

(b) 1

(c) -1

(d) 0

(e) $2/\pi$

15.) Compute $\displaystyle\lim_{x \to 0^+} 2\sqrt{x} \ln x$.

(a) Does not exist.

(b) 0

(c) 4

(d) 8

(e) -4

Place your answers in the boxes provided.
You must show your work to receive credit.

16.) As a (spherical) balloon is inflated its surface area increases at a rate of 5 cm^2/sec. How rapidly is the volume of the balloon increasing at the time when the radius is 14 cm?

17.) Find the points(s) on the hyperbola $y^2 - 3x^2 = 6$ that are closest to the point $(4, 0)$.

18.) Differentiate the following functions.

(a) $f(x) = \dfrac{e^{x^2}\tan(x)}{x+\sqrt{x}}$.

(b) $g(t) = \sin(\ln(\pi\cos(t))$

(c) $h(x) = \displaystyle\int_{-99}^{\sqrt[3]{x}} \frac{e^t + e^{\sqrt{t}}}{\sec(t)}\,\mathrm{d}t$.

19.) Compute the following integrals.

(a) $\displaystyle\int_1^4 \left(x^5 + \sqrt{x} - 7\right)\, \mathrm{d}x.$

(b) $\displaystyle\int_{-\pi}^{3\pi/2} \cos\left(3t - \pi\right)\, \mathrm{d}t.$

(c) $\displaystyle\int \frac{x^2}{(1-x)^2}\mathrm{d}x.$

(d) $\displaystyle\int \frac{x}{1+x^4}\,\mathrm{d}x.$

20.) Sketch the parabola $y = x^2 - 2x$; plot the point $(-3, 6)$. Find the equation(s) of the line(s) that are tangent to the parabola and pass through the point $(-3, 6)$.