

## PART 1

Mark your answer on the scantron. (Since your scantrons are not returned, we suggest that you also circle the answer below.) Each question is worth 4 points.

1.) Compute  $\lim_{x \rightarrow 0} \frac{e^{3x} - 1}{\sin(2x)}$ .  $\rightarrow \frac{0}{0}$ . Apply L'Hospital

(a) 0

(b)  $\frac{9}{2}$

(c)  $\frac{3}{2}$

(d)  $\frac{3}{2e}$

(e) does not exist.

$$\frac{3e^{3x}}{2 \cos(2x)} \rightarrow \frac{3}{2}$$

2.) A particle moving with velocity  $\mathbf{v}(t) = \langle \sqrt[3]{t}, t^3 \rangle$  has position vector  $\langle 1, -1 \rangle$  at time  $t = 0$ . Find the position vector of a particle at time  $t = 1$ .

(a)  $\langle 2, 0 \rangle$

(b)  $\langle 1, 1 \rangle$

(c)  $\langle \frac{4}{3}, 2 \rangle$

(d)  $\langle \frac{1}{3}, 3 \rangle$

(e)  $\langle \frac{7}{4}, -\frac{3}{4} \rangle$

$$\mathbf{v} = \langle t^{1/3}, t^3 \rangle = \mathbf{r}'$$

$$\Rightarrow \mathbf{r} = \langle \frac{3}{4}t^{4/3} + C_1, \frac{1}{4}t^4 + C_2 \rangle$$

$$\langle 1, -1 \rangle = \mathbf{r}(0) = \langle C_1, C_2 \rangle$$

$$\Rightarrow \mathbf{r}(1) = \langle \frac{3}{4} + 1, \frac{1}{4} - 1 \rangle$$

3.) Find all critical numbers of  $f(x) = x^5 - 5x^4 + 5x^3$ .

(a)  $x = 0, 1, 3$

(b)  $x = 0, \frac{5}{2} \pm \frac{3}{2}\sqrt{5}$

(c)  $x = 0, \frac{3}{2} \pm \frac{1}{2}\sqrt{3}$

(d)  $x = 1, 3$

(e)  $x = 0$ .

$$f' = 5x^4 - 20x^3 + 15x^2$$

$$f' = 5x^2(x^2 - 4x + 3)$$

$$f' = 5x^2(x-3)(x-1)$$

4.) Compute the derivative of  $f(x) = \ln(\sin^{-1} x) = \ln(\arcsin x)$ .

(a)  $f'(x) = -\cot x$

(b)  $f'(x) = \frac{1}{x\sqrt{1-x^2}}$

(c)  $f'(x) = \frac{1}{\sin^{-1}(x)\sqrt{1-x^2}}$

(d)  $f'(x) = \frac{\sin x}{\sqrt{1-x^2}}$

(e)  $f'(x) = \sqrt{1-x^2}$ .

5.) Determine the interval(s) on which the function  $f(x) = x^5 - 15x^3$  is increasing.

(a)  $(-\infty, -3)$  and  $(3, \infty)$

(b)  $(-3, 0)$  and  $(3, \infty)$

(c)  $(-\infty, -3)$  and  $(0, 3)$

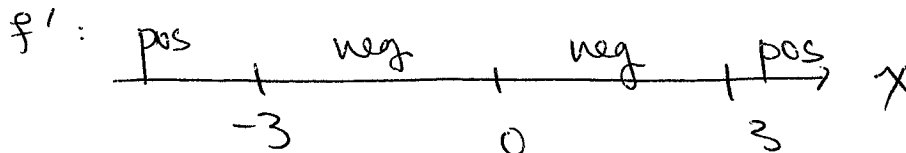
(d)  $(-3, 3)$

(e)  $(0, \infty)$ .

$f' = 5x^4 - 45x^2$

$= 5x^2(x^2 - 9)$

Crit. numbs  $x = 0, \pm 3$



6.) Compute  $\lim_{x \rightarrow \infty} \frac{\ln(3x)}{x} \rightarrow \frac{\infty}{\infty}$

(a) 3

(b) 1

(c)  $\ln 3$

(d) 0

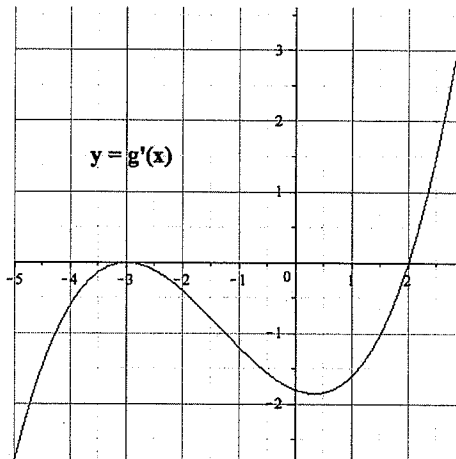
(e) does not exist.

Apply L'Hospital  $\frac{(\frac{3}{3x})}{1} \rightarrow 0$

To the right is a graph

$$y = g'(x)$$

of the derivative. Use this graph to answer the following three questions about the function  $g(x)$ .



- 4.) [Use the graph of  $y = g'(x)$  above.] The inflection points of  $y = g(x)$  occur at
- (a)  $x = -3, 2$
  - (b)  $x = -3, 0.4$
  - (c)  $x = -3, 0.4, 2$
  - (d)  $x = -1.5$
  - (e)  $x = -3, -1.5, 0.4, 2$ .
- 5.) [Use the graph of  $y = g'(x)$  above.] The graph of  $y = g(x)$  is concave up on the interval(s)
- (a)  $(-3, 0.4)$
  - (b)  $(-5, -3)$  and  $(2, 3)$
  - (c)  $(-1.5, 3)$
  - (d)  $(-3, 2)$
  - (e)  $(-5, 3)$  and  $(0.4, 3)$ .
- 6.) [Use the graph of  $y = g'(x)$  above.] The function  $y = g(x)$  has a local minimum at
- (a)  $x = 2$
  - (b)  $x = 0.4$
  - (c)  $x = -3$
  - (d)  $x = -3, 2$
  - (e)  $x = -5$ .

7.) The anti-derivative of  $f(x) = x^e + e^x$  is: (Here  $e = 2.7182818\dots$  is the familiar base of our favorite exponential function.)

(a)  $ex^{e-1} + e^x + C$

(b)  $x^e \ln(x) + xe^{x-1} + C$

(c)  $\frac{x^{e+1}}{e+1} + e^x + C$

(d)  $\frac{x^e}{\ln x} + \frac{e^{x+1}}{x+1} + C$

(e)  $\ln(x^e + e^x) + C$

8.) Compute  $\sum_{k=1}^{10} (3k - 20)$ .  $= 3\left(\sum_1^{10} k\right) - 20\left(\sum_1^{10} 1\right)$

(a) -170

(b) 10

(c) 100

(d) -35

(e) -70.

$= 3 \frac{1}{2} (10)(11) - 200$

$= 165 - 200$

9.) A rectangular area of  $30,000 \text{ ft}^2$  is to be fenced in. One side requires expensive fencing that costs  $\$5/\text{ft}$ . The fencing for the remaining three sides is a less expensive  $\$3/\text{ft}$ . If the total cost is minimized, how long is the expensive ( $\$5/\text{ft}$ ) fencing?

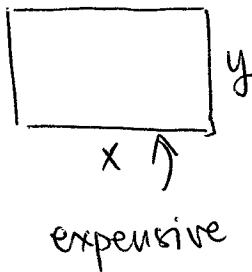
(a) 50 ft

(b)  $100\sqrt{3}$  ft

(c)  $100\sqrt{6}$  ft

(d) 150 ft

(e) 0 ft.



$30000 = xy$

$C = 5x + 3(x+2y)$

$C = 8x + \frac{180,000}{x}$

$C' = 8 - \frac{180,000}{x^2}$

$x = \sqrt{\frac{180,000}{8}} = \sqrt{22500}$

## PART 2

Place your answers in the boxes provided.  
You must show your work to receive credit.

- 13.) A radioactive isotope decays exponentially. The initial mass of the isotope is 729 grams. After 100 years the mass has decayed to 81 grams. [8 points]

a.) What is the mass after 150 years?

27 grams

$$729 = 9^3 \\ = 27^2$$

$$M(t) = 729 e^{-kt}$$

$$81 = 729 e^{-k100}$$

$$\ln\left(\frac{1}{9}\right) = -k100$$

$$\frac{\ln(9)}{100} = k$$

$$\ln\left(9^{t/100}\right) = kt$$

$$M(t) = 729 e^{-\ln(9^{1/100})kt} \\ = 729 \left(\frac{1}{9}\right)^{t/100}$$

$$M(150) = 729 \left(\frac{1}{9}\right)^{3/2}$$

$$= 729 \left(\frac{1}{27}\right) = 27 \text{ grams}$$

b.) What is the half-life of the isotope?

$100 \frac{\ln(2)}{\ln(9)}$  years.

$$\frac{1}{2} = \left(\frac{1}{9}\right)^{t/100}$$

$$\ln(2) = \frac{t}{100} \ln(9)$$

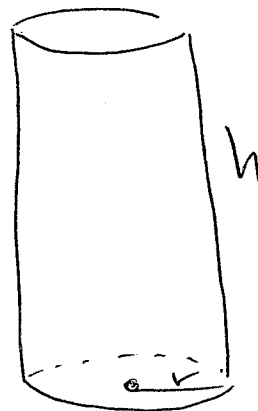
$$t = 100 \frac{\ln(2)}{\ln(9)}$$

- 14.) A cylindrical tank with **no lid** is constructed from  $300\pi m^2$  of material. Find the radius and height that will maximize the volume of the tank. [12 points]

$$300\pi = \pi r^2 + 2\pi r h$$

$$V = \pi r^2 h$$

$$\frac{300 - r^2}{2r} = h$$



$$V = \pi r \frac{300 - r^2}{2} = \frac{\pi}{2} (300r - r^3)$$

$$V' = \frac{\pi}{2} (300 - 3r^2) \Rightarrow r = \pm 10$$

radius is nonnegative  $\Rightarrow r = 10$

$$h = \frac{300 - 100}{20} = 10$$

$r = h = 10 \text{ meters.}$

15.) Compute the derivatives of the following functions. [6 points each]

a.)  $f(x) = \arctan(\sqrt{x^2+1})$ .

$$\frac{1}{1+(1+x^2)} \cdot \frac{2x}{2\sqrt{x^2+1}}$$

$$\frac{1}{2+x^2} \cdot \frac{x}{\sqrt{x^2+1}}$$

b.)  $f(x) = (x^2+x+1)^{\ln x}$ .

Diff'iate  
↳

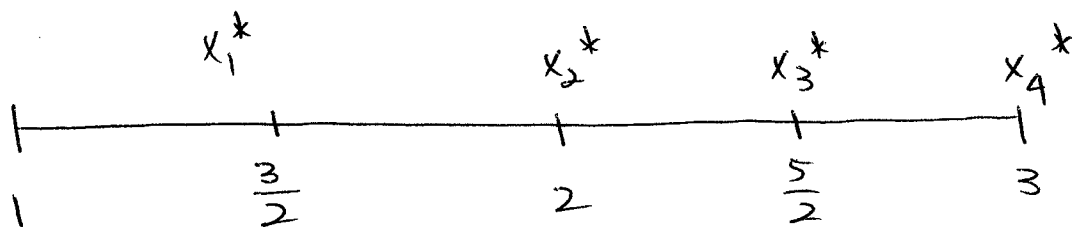
$$\ln f = (\ln x) \ln(x^2+x+1)$$

Apply ln

$$\frac{f'}{f} = \frac{1}{x} \ln(x^2+x+1) + (\ln x) \frac{2x+1}{x^2+x+1}$$

$$f' = (x^2+x+1)^{\ln x} \left\{ \frac{\ln(x^2+x+1)}{x} + \frac{(\ln x)(2x+1)}{x^2+x+1} \right\}$$

- 16.) Partition the interval  $1 \leq x \leq 3$  into four subintervals (of equal length). Use right-endpoints to approximate the area between the curve  $y = 1/x$  and the interval. [8 points]



$$\text{Area} \approx \frac{1}{2} \left( \frac{2}{3} + \frac{1}{2} + \frac{2}{5} + \frac{1}{3} \right)$$

||

$$\frac{1}{2} \left( \frac{4+3+15+2}{6} \right) = \frac{24}{12} = 2$$

$$\begin{aligned} \text{Area} &\approx \frac{1}{2} \left( \frac{20+15+12+10}{30} \right) = \frac{57}{60} \\ &= \frac{19}{20} \end{aligned}$$

$\text{Area} \approx \frac{19}{20}$

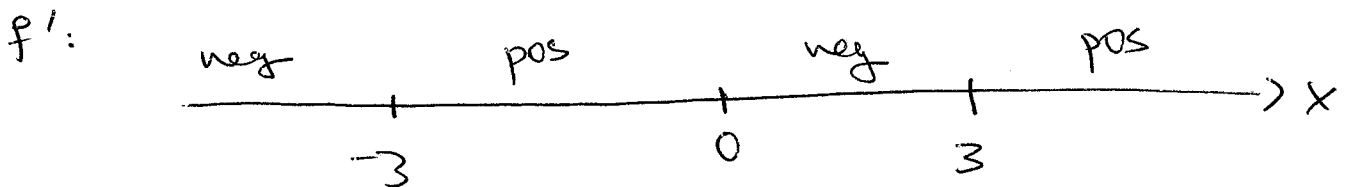
17.) Set  $f(x) = \frac{1}{9}(x^4 - 18x^2)$ . [12 points]

a.) Find the critical numbers of  $f(x)$ .

$$f' = \frac{1}{9}(4x^3 - 36x) = \frac{x}{9}(4x^2 - 36)$$

$$x=0, \pm 3$$

b.) Find the intervals of increase and decrease for  $f(x)$ .



$$\begin{aligned} \text{INC: } & (-3, 0) \cup (3, \infty) \\ \text{DEC: } & (-\infty, -3) \cup (0, 3) \end{aligned}$$

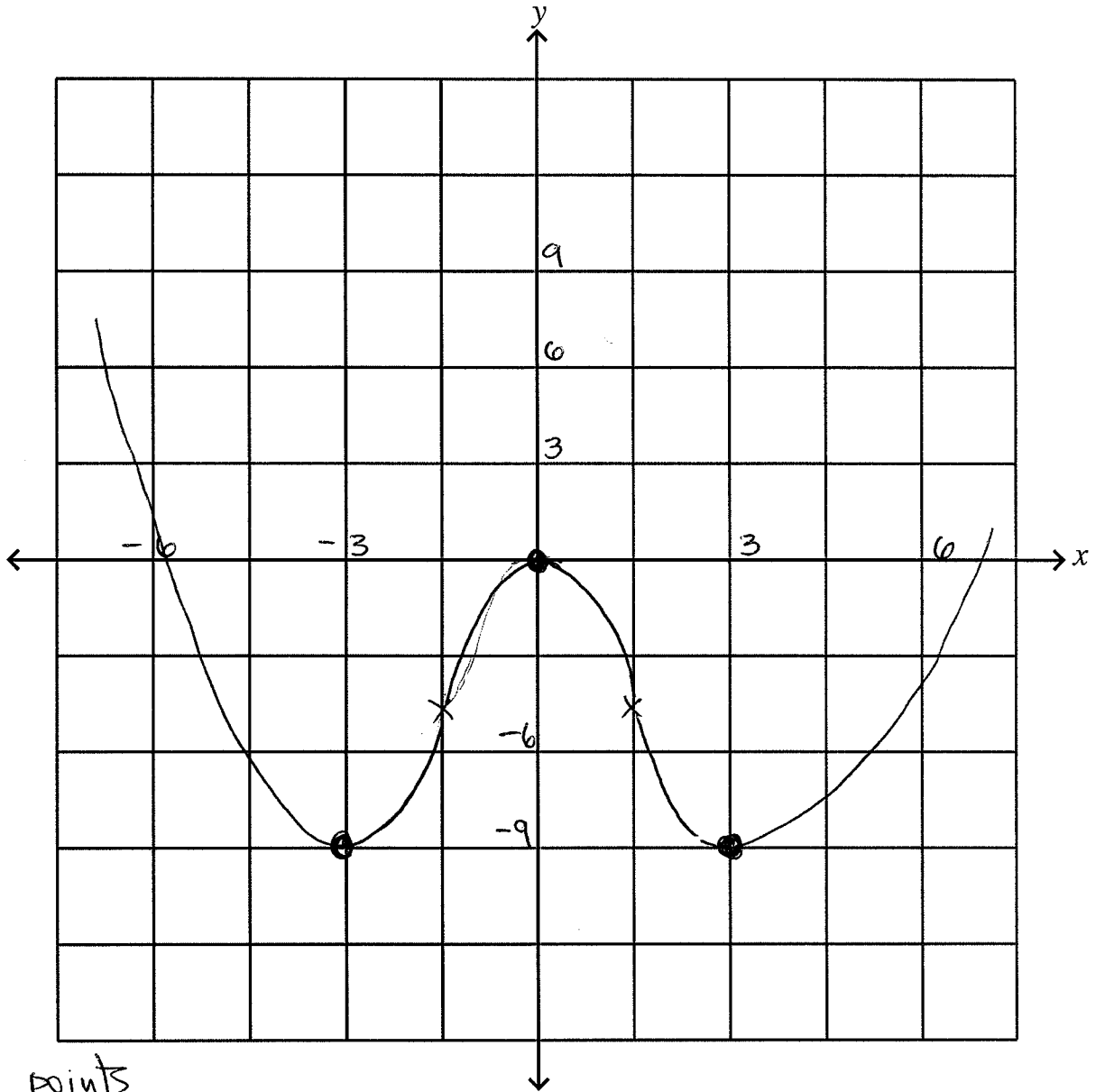
c.) Find the intervals of concavity for  $f(x)$ .

$$f'' = \frac{1}{9}(12x^2 - 36) = \frac{1}{3}(4x^2 - 9)$$

$$\text{IP} = \pm \frac{3}{2}$$

$$\begin{aligned} \text{C.UP: } & (-\infty, -3/2) \cup (3/2, \infty) \\ \text{C.DOWN: } & (-3/2, 3/2) \end{aligned}$$

d.) Sketch the graph  $y = f(x)$ . Be sure that your sketch accurately reflects the critical points, intervals of increase/decrease and concavity identified above.



critical points

$$f(0) = 0$$

$$f(\pm 3) = 9 - 18 = -9$$