Practice Test Solutions

Q1
$$\left(\begin{array}{ccc|c} 1 & -2 & 1 & 1 \\ 1 & -3 & 2 & 0 \\ 3 & -8 & 5 & a \end{array}\right) \to \left(\begin{array}{ccc|c} 1 & -2 & 1 & 1 \\ 0 & -1 & 1 & -1 \\ 0 & -2 & 2 & a-3 \end{array}\right)$$

$$\to \left(\begin{array}{ccc|c} 1 & -2 & 1 & 1 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & 0 & a-1 \end{array}\right)$$

so we must have $a = 1$.

The solution is $\begin{pmatrix} 3+t \\ 1+t \\ t \end{pmatrix}$

Q2
$$\begin{pmatrix} 1 & 2 & 3 & 1 & 0 & 0 \\ 1 & 3 & 5 & 0 & 1 & 0 \\ 2 & 4 & 7 & 0 & 0 & 1 \end{pmatrix} \to \begin{pmatrix} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & 2 & -1 & 1 & 0 \\ 0 & 0 & 1 & -2 & 0 & 1 \end{pmatrix}$$

$$\to \begin{pmatrix} 1 & 0 & -1 & 3 & -2 & 0 \\ 0 & 1 & 2 & -1 & 1 & 0 \\ 0 & 0 & 1 & -2 & 0 & 1 \end{pmatrix} \to \begin{pmatrix} 1 & 0 & 0 & 1 & -2 & 1 \\ 0 & 1 & 0 & 3 & 1 & -2 \\ 0 & 0 & 1 & -2 & 0 & 1 \end{pmatrix}$$

Inverse is $\begin{pmatrix} 1 & -2 & 1 \\ 3 & 1 & -2 \\ -2 & 0 & 1 \end{pmatrix}$

Q3 On $[0,1]$ $|x| = x$ so l.d.

On $[-1,1]$ if $\alpha_1 x + \alpha_2 |x| = 0$ then

$\alpha_1 + \alpha_2 = 0$ on $[0,1]$ and

$\alpha_1 - \alpha_2 = 0$ on $[-1,0]$ so $\alpha_1 = \alpha_2 = 0$.

l.i.

Q4 $\alpha_1(x+5)+\alpha_2(x+6) = 3x+2$

$\underline{x}$ $\quad \alpha_1+\alpha_2 = 3$

$\underline{\text{const}}$ $\quad 5\alpha_1+6\alpha_2 = 2$

$$\left(\begin{array}{cc|c} 1 & 1 & 3 \\ 5 & 6 & 2 \end{array}\right) \to \left(\begin{array}{cc|c} 1 & 1 & 3 \\ 0 & 1 & -13 \end{array}\right)$$

$$\Rightarrow \alpha_2 = -13, \quad \alpha_1 = 16$$

Q5
$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 4 & 1 \\ 1 & 0 & 1 & 1 \\ 2 & 1 & 6 & 6 \end{pmatrix} \to \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 4 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & 6 & 6 \end{pmatrix} \to \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 4 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 2 & 5 \end{pmatrix}$$

$$\to \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 4 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 3 \end{pmatrix} \cdots \to \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{pmatrix}$$

Null space $= \{0\}$

Q6 $\alpha_1(x^2+x-2) + \alpha_2(x^2+2x-3) + \alpha_3(x^2+3x-4) = a_0+a_1x+a_2x^2$

$\underline{x^2}$ $\quad \alpha_1+\alpha_2+\alpha_3 = a_2$

$\underline{x}$ $\quad \alpha_1+2\alpha_2+3\alpha_3 = a_1$

$\underline{\text{const}}$ $\quad -2\alpha_1-3\alpha_3-4\alpha_3 = a_0$

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & a_2 \\ 1 & 2 & 3 & a_1 \\ -2 & -3 & -4 & a_0 \end{array}\right) \to \left(\begin{array}{ccc|c} 1 & 1 & 1 & a_2 \\ 0 & 1 & 2 & a_1 \\ 0 & -1 & -2 & a_0-2a_2 \end{array}\right)$$

$$\to \left(\begin{array}{ccc|c} 1 & 1 & 1 & a_2 \\ 0 & 1 & 2 & a_1 \\ 0 & 0 & 0 & a_0-2a_2+a_1 \end{array}\right)$$

Cannot always solve so not spanning!

Q7 No $\begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix} + \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 2 & 2 \end{pmatrix}$

$\in W \qquad \in W \qquad \notin W.$

Q8 N/A

Q9 If $f(0) = 2f(1)$ and $g(0) = 2g(1)$

then $(f+g)(0) = f(0) + g(0) = 2f(1) + 2g(1)$

$= 2\left((f+g)(1)\right)$. Closed under addition.

If $\alpha \in \mathbb{R}$ then $(\alpha f)(0) = \alpha f(0) = \alpha 2 f(1)$

$= \cancel{\alpha(2f} \; 2(\alpha f)(1)$

Closed under scalar mult.

Yes a subspace.

Q10 Solve $\left(\begin{array}{cccc|c} 1 & 2 & 3 & 4 & a \\ 2 & 5 & 7 & 9 & b \\ 3 & 5 & 8 & 12 & c \end{array}\right)$

$\rightarrow \left(\begin{array}{cccc|c} 1 & 2 & 3 & 4 & a \\ 0 & 1 & 1 & 1 & b-2a \\ 0 & -1 & -1 & 0 & c-3a \end{array}\right)$

$\rightarrow \left(\begin{array}{cccc|c} 1 & 2 & 3 & 4 & a \\ 0 & 1 & 1 & 0 & 3a-c \\ 0 & 1 & 1 & 1 & b-2a \end{array}\right)$

$\rightarrow \left(\begin{array}{ccccc} 1 & 2 & 3 & 4 & a \\ 0 & 1 & 1 & 0 & 3a-c \\ 0 & 0 & 0 & 1 & b-2a-(3a-c) \end{array}\right)$ can always solve.

Not l.i., two many vectors