

MATH 304, TEST 1

Show all steps for credit. 10 pts. per question

Q1. Find the value of a that makes the equations

$$\begin{aligned}x_1 - 2x_2 + x_3 &= 1 \\x_1 - 3x_2 + 2x_3 &= 0 \\3x_1 - 8x_2 + 5x_3 &= a\end{aligned}$$

consistent, and solve them for this value of a .

Q2. Find the inverse of the matrix

$$\begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 5 \\ 2 & 4 & 7 \end{pmatrix}.$$

Q3. Are the functions $\{x, |x|\}$ linearly independent in $C[0, 1]$? What about in $C[-1, 1]$?

Q4. Find the vector $3x + 2$ in P_2 as a linear combination of the vectors $x + 5$ and $x + 6$.

Q5. Put the following matrix into *reduced* row echelon form:

$$A = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 4 & 1 \\ 1 & 0 & 1 & 1 \\ 2 & 1 & 6 & 6 \end{pmatrix}.$$

Now find its null space $N(A)$.

Q6. Is $\{x^2 + x - 2, x^2 + 2x - 3, x^2 + 3x - 4\}$ a spanning set for P_3 ?

Q7. In the vector space $\mathbb{R}^{2 \times 2}$ of 2×2 matrices, is

$$W = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} : abcd = 0 \right\}$$

a subspace?

Q8. A and B are 2×2 matrices with

$$\det(A) = 3, \quad \det(B) = 7.$$

Find the determinants of

$$AB, \quad A^{-1}B, \quad 2B, \quad 3A^{-1}, \quad B^{-1}A^T.$$

Q9. In the vector space $C[0, 1]$ of continuous functions on $[0, 1]$, let

$$W = \{f : f(0) = 2f(1)\}.$$

Is W a subspace?

Q10. Do the vectors

$$\left\{ \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \begin{pmatrix} 2 \\ 5 \\ 5 \end{pmatrix}, \begin{pmatrix} 3 \\ 7 \\ 8 \end{pmatrix}, \begin{pmatrix} 4 \\ 9 \\ 12 \end{pmatrix} \right\}$$

span \mathbb{R}^3 ? Are they linearly independent?