MATH 446, PRACTICE 2nd TEST

20 pts per question. Name theorems you use or state them.
Honors: *do all problems.* **Non-honors**: *do* **any five** *problems*

Q1. Prove that $\operatorname{span}\{1, x^2, x^4, \ldots\}$ is uniformly dense in $C[0,1]$. What is its uniform closure in $C[-1,1]$?

Q2. (a) If $f \in C[0,1]$ is given and $A = \operatorname{span}\{1, f, f^2, f^3, \ldots\}$, prove that $\overline{A} = C[0,1]$ if and only if f separates points in $[0,1]$.

(b) Assuming that f *does* separate points, prove that $\overline{B} = C[0,1]$ where $B = \operatorname{span}\{1, g, g^2, g^3, \ldots\}$ and $g(x) = f(x^3)$.

Q3. Prove that there is no continuous one to one onto map of the square to the circle (consider removing 2 points)

Q4. Define $T : C[0,1] \to C[0,1]$ by

$$Tf(x) = \int_0^{x^2} f(t)\,dt.$$

Prove that T^2 is a contraction.

Q5. Prove that a monotone function on $[0,1]$ is Riemann integrable.

Q6. Let $f(x) = 1 + \arctan x$. Find an interval $[-c, c]$ which f maps to itself. Prove that the equation $2x = f(x)$ has a unique solution in this interval.