MATH607 SEVENTH HOMEWORK

The results on dense sets in $L^1$ are worth remembering.

Hand in the first 3 questions

Q1. Let $f \in L^1(\mathbb{R})$ with Lebesgue measure. Prove that

$$\lim_{n \to \infty} \int f(x) \cos(nx) \, dx = 0.$$ 

Q2. Let $f \in L^1(\mathbb{R})$ with Lebesgue measure $\lambda$. Given $\varepsilon > 0$, prove that there exists $\delta > 0$ such that

$$\int_A |f| < \varepsilon \quad \text{whenever} \quad \lambda(A) < \delta, \ A \text{ measurable}.$$ 

Q3. Let $f \in L^1(\mathbb{R})$ with Lebesgue measure. Prove that

$$\lim_{h \to 0} \int |f(x+h) - f(x)| \, dx = 0.$$ 

Q4. If $f : [0,1] \to \mathbb{R}^+$ is Lebesgue measurable, find all possibilities for

$$\lim_{n \to \infty} \int_0^1 f^n.$$ 

Q5. If $f : [0,1] \to \mathbb{R}^+$ is in $L^1(\mathbb{R})$, find

$$\lim_{n \to \infty} \int_0^1 f^{1/n}.$$