SOLUTIONS

TEST 1

Q1

A = "All gets 2 aces"
B = "Bob gets 2 aces"

\[ P(\text{A} \cup \text{B}) = P(\text{A}) + P(\text{B}) - P(\text{A} \cap \text{B}), \]

\[ P(\text{A}) = \frac{\binom{4}{2}}{\binom{52}{2}}, \text{ and } P(\text{B}) \text{ is the same.} \]

\[ P(\text{A} \cap \text{B}) \text{ means that the 4 cards given out were all aces, so } P(\text{A} \cap \text{B}) = \frac{1}{\binom{52}{4}} \]

\[ P(\text{A} \cup \text{B}) = 2 \frac{\binom{4}{2}}{\binom{52}{2}} - \frac{1}{\binom{52}{2} \cdot \binom{52}{4}} \]

Q2

\[ \text{Prob} = \frac{2}{3} \cdot \frac{7}{11} + \frac{1}{3} \cdot \frac{43}{11} = \frac{14}{33} + \frac{43}{33} = \frac{57}{33} = \frac{17}{33} \]
Q3. A committee can violate the rule if it has

a) all men : 5 choices
b) all women : 5 choices
c) 2 couples : \( \binom{5}{2} = 10 \) choices

The number of committees is \( \binom{10}{4} = \frac{10 \cdot 9 \cdot 8 \cdot 7}{4 \cdot 3 \cdot 2} = 210 \)

\[ P(\text{rule is violated}) = \frac{5 + 5 + 10}{210} = \frac{2}{3} \]

Q4. B = "2 black" W = "2 white" R = "2 red"

M = "they match". We want \( P(B|M) \).

This is \( \frac{P(M|B) \cdot P(B)}{P(M)} = \frac{P(B)}{P(M)} \) since \( P(M|B) = 1 \).

\[ P(B) = \frac{\binom{6}{4,2}}{\binom{14}{4,2}} = \frac{15}{91} \]

\[ P(W) = \frac{\binom{4}{4,2}}{\binom{14}{4,2}} = \frac{6}{91} \text{ and } P(R) = \frac{6}{91} \]

\[ \therefore P(B|M) = \frac{15}{15 + 6 + 6} = \frac{15}{27} = \frac{5}{9} \]
45. (i) With the ace of spades
   Need 2 more aces: \( C_{3,2} \)
   Need 1 more spade: \( C_{12,1} \)
   Need 1 non-spade non-ace: \( C_{36,1} \)

(ii) Without the ace of spades:
   Need all the other aces: (1 choice)
   Need 2 non-ace spades: \( C_{12,2} \)

Total: \( C_{3,2} C_{12,1} C_{36,1} + C_{12,2} \)

46. \( p = \) prob. Al wins. Condition on the first round.

\[
p = \frac{1}{6} \cdot \frac{1}{2} + \frac{5}{6} \cdot \frac{1}{2} \cdot p
\]

\( \uparrow \quad \uparrow \quad \uparrow \)

\( \text{S(x) tail non-ace tail} \)

\[
12p = 1 + 5p, \quad 7p = 1, \quad p = \frac{1}{7}
\]
Q7

\[
P(X=3) = \frac{1^3 e^{-1}}{3!} = \frac{e^{-2}}{6} = \frac{4}{3 e^2} = 0.1804
\]

Q8

3 cases.

a) pick 2R with prob \( \frac{1}{C_{4,2}} = \frac{1}{6} \). Now we have

\# 2 has 4R, 2R and we must pick 2R so

\[
\frac{C_{4,2}}{C_{6,2}} = \frac{6}{15} = \frac{2}{5}.

\text{Prob for a) is } \frac{1}{6} \cdot \frac{2}{5} = \frac{1}{15}
\]

b) pick 2B. Same as a) \( \frac{1}{15} \)

c) pick 1R 1B with prob. \( \frac{C_{2,1} C_{3,1}}{C_{4,2}} = \frac{4}{6} = \frac{2}{3} \)

Now \# 2 has 3R, 3B and we pick 1R, 1B with prob

\[
\frac{C_{3,1} C_{3,1}}{C_{6,2}} = \frac{9}{15} = \frac{3}{5}.

\text{Prob for c) is } \frac{2}{3} \cdot \frac{3}{5} = \frac{2}{5}
\]

\[
\text{Total} = \frac{1}{15} + \frac{1}{15} + \frac{6}{15} = \frac{8}{15}
\]
\[ P(A_1 \cup A_2 \cup A_3 \cup A_4) = \sum P(A_i) - \sum P(A_i \cap A_j) + \sum P(A_i \cap A_j \cap A_k) - P(A_i \cap A_j \cap A_k \cap A_l) \]

\[ A_1 = \text{"no spades"} \quad A_2 = \text{"no hearts"} \quad A_3 = \text{"no diamonds"} \quad A_4 = \text{"no clubs"} \]

\[ P(A_1 \cup A_2 \cup A_3 \cup A_4) = 4 \cdot P(A_1) - 6 \cdot P(A_1 \cap A_2) + 4 \cdot P(A_1 \cap A_2 \cap A_3) \]

\[ = \frac{4 \cdot C_{39, 13} - 6 \cdot C_{26, 13} + 4 \cdot C_{13, 13}}{C_{52, 13}} \]

Q10.

a) no pairs. Choose 4 colors \( C_{6, 4} = 15 \). Let \( k \) be right for each color \( 2^k \).

\[ P(X = 0) = \frac{15 \cdot 16}{C_{12, 4}} = \frac{15 \cdot 16 \cdot 4 \cdot 3 \cdot 2}{12 \cdot 11 \cdot 10 \cdot 9} = \frac{16}{33} \]

b) 1 pair. Choose the pair, 6 choices. Choose 2 colors \( C_{5, 2} \).

\[ \text{left or right } 2^2 = 4, \quad P(X = 1) = \frac{6 \cdot 10 \cdot 4 \cdot 3 \cdot 2}{12 \cdot 11 \cdot 10 \cdot 9} = \frac{16}{33} \]

c) 2 pairs. \[ P(X = 2) = \frac{C_{6, 2}}{C_{12, 4}} = \frac{15}{495} = \frac{1}{33} \]

\[ E(X) = 0 \cdot \frac{16}{33} + 1 \cdot \frac{16}{33} + 2 \cdot \frac{1}{33} = \frac{18}{33} = \frac{6}{11} \]