Q1. Three distinct awards are to be given to a group of five students. In how many ways can this be done if

(i) no student receives more that one award

(ii) no student receives more than two awards.

Q2. 5 men are wearing red shirts and 5 men are wearing blue shirts. If the 10 men are lined up randomly, what is the probability that the colors alternate?

Q3. Urn #1 contains 3 red balls and 1 blue ball, urn #2 contains 4 red balls and 2 blue balls. Two balls are picked from one of the urns and observed to be red. What is the probability that they came from urn #1?

Q4. A cereal company puts a toy soldier in 10% of its boxes and a toy sailor in 90% of its boxes. If you open boxes repeatedly, what is the probability that the \( n \)th box is the first time that you have seen both toys. What is the expected number of boxes that you must open to get both toys? (Remember 1/(1 – \( x \)) = 1 + \( x \) + \( x^2 \) + . . .)

Q5. Ten couples go to a restaurant and the twenty people are randomly seated at 5 four-person tables. What is the expected number of men who sit at the same table as their wives?

Q6. State the inclusion-exclusion formula for four events.

If 13 cards are drawn from a pack of 52, how many such hands contain a four-card suit (i.e. exactly four spades or exactly four hearts etc.)? Leave your answer in terms of the \( C_{n,k} \)'s.
Q7. Two cards are drawn from a pack of 52. The first is a heart, the second is an ace. Are these independent events?

Q8. X and Y are independent random variables each with uniform density on [0, 1]. Find the distribution and density functions for Z = X^2Y.

Q9. Random variables X and Y have joint density function

\[ f_{X,Y}(x, y) = \begin{cases} 2x, & 0 \leq x \leq 1, \ 0 \leq y \leq 1, \\ 0, & \text{otherwise}. \end{cases} \]

Let Z = \( \min\{X, Y\} \). Find \( f_Z(z) \) and \( E(Z) \).

Q10. A random variable X has mean 20 and variance 10. Use Chebyshev’s inequality to estimate \( P(10 < X < 30) \).

Q11. Alice owns 10 distinct pairs of shoes and a bad dog. One day the dog randomly chooses 5 of her shoes and destroys them. What is the expected number of pairs of shoes that survive this event?

Q12. Every evening Jim and Mary go to one of three types of restaurants, Italian, French or Chinese. They never eat at the same type on two consecutive evenings. If they eat Italian then they eat French the next evening, if they eat French then they switch to Italian or Chinese with equal probability, while if they eat Chinese then they switch to Italian with probability 0.2. In the long run, what proportion of their evenings are spent in Italian restaurants?