Homework #1 Solutions

1.7
1. There are 6. ABC, ACB, BAC, BCA, CBA, CBA

2. (i) Any number 0, ..., 9 are equally likely with prob \( \frac{1}{10} \)
    (ii) Numbers don't begin with 0 so
        1, ..., 9 occur with prob \( \frac{1}{9} \).

4. Of the 36 possibilities 6 give equal scores
    so 30 give unequal scores. In half of these,
    15, red will be larger than green.

5. (1) If the first number is 1 or 6 there are
    2 possibilities for each, (1,1), (1,2), (6,6), (6,5).
    For the other numbers, there are 3 possibilities,
    (n,n), (n,n+1), (n,n-1) for a total of
    \[ 2 + 2 + 3 + 3 + 3 = 16 \quad \text{Prob} = \frac{16}{36} = \frac{4}{9} \]

(2) If the move is 5, there are 9 possibilities
    (5,1), (5,2), (5,3), (5,4), (5,5), (4,5), (3,5), (2,5), (1,5).

If the move is 6 there are 11 possibilities
(6,1), (6,2), (6,3), (6,4), (6,5), (6,6), (5,6), (4,6), (3,6), (2,6), (1,6)

\[ \text{Prob} = \frac{9 + 11}{36} = \frac{20}{36} = \frac{5}{9} \]
6. We can get 0, 2, 4 heads

0 heads  All tails  \( \frac{1}{32} \)

2 heads  10 choices for the position of the heads so

\( \frac{10}{32} \)

4 heads  5 choices for the position of the tail so

\( \frac{5}{32} \).

\[ \text{Prob (even)} = \frac{1+10+5}{32} = \frac{16}{32} = \frac{1}{2}. \]

7. Assume they play the last three games.

1. A A A
2. A B A
3. A A B
4. A B B
5. B B B
6. B A B
7. B B A
8. B A A

The winner of game 5 wins overall in

\[ 1, 2, 3, 5, 6, 7 \]

\[ \rightarrow \text{Prob} = \frac{6}{8} = \frac{3}{4}. \]
9. There are \( 4 + 16 + 15 + 12 = 47 \) children.

Prob of coming from families of 1, 2, 3 or 4 children are

\[ \frac{4}{47}, \frac{16}{47}, \frac{15}{47}, \frac{12}{47}. \]

(11) We add together the second and third dice. As in class, this table gives the no. of times a score is achieved:

<table>
<thead>
<tr>
<th>Score</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>4</td>
<td>5</td>
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<td>5</td>
<td>6</td>
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<td>6</td>
<td>7</td>
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<td>8</td>
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<td>8</td>
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<td>9</td>
<td>10</td>
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<tr>
<td>10</td>
<td>11</td>
</tr>
<tr>
<td>11</td>
<td>12</td>
</tr>
<tr>
<td>12</td>
<td>1</td>
</tr>
</tbody>
</table>

We can get 7 as

1 + 6 (5 times)
2 + 5 (4 times)
3 + 4 (3 times)
4 + 3 (2 times)
5 + 2 (1 time)
6 + 1 (impossible)

Prob. = \[ \frac{15}{6^3} = \frac{5}{2\cdot 36} = \frac{5}{72} \]

(f) We can get 8 as 1 + 7 (6 times)
2 + 6 (5 times) 3 + 5 (4 times) 4 + 4 (3 times)
5 + 3 (2 times) 6 + 2 (1 time)

no. Prob. = \[ \frac{21}{6^3} = \frac{7}{2\cdot 6^2} = \frac{7}{72} \]
12. \[ S \cap D \]

\[
10 \quad 15 \quad 45
\]

\[ \text{Ans} = 10 + 15 + 45 = 70 \%
\]

17. \[ A \quad B \]

\[ P(A \cup B) = 0.8 \text{ by disjointness} \]

\[ P(A^c \cap B^c) = P((A \cup B)^c) = 1 - 0.8 = 0.2 \]