20. (i) $A = \text{at most 1 girl.}$

$P(A) = P(0 \text{ girl}) + P(1 \text{ girl})$

$= \frac{1}{8} + \frac{3}{8} = \frac{1}{2}$

$B = \text{children of both sexes}$

$P(B) = 1 - (P(\text{no girl}) + P(\text{no boy}))$

$= 1 - \frac{1}{8} - \frac{1}{8} = \frac{3}{4}$

$A \cap B = \text{"1 girl"}$

$P(A \cap B) = \frac{3}{8} = P(A) \cdot P(B)$ so independent.

(ii) With 4 children

$P(A) = \frac{1}{16} + \frac{4}{16} = \frac{5}{16}$

$P(B) = 1 - \left(\frac{1}{16} + \frac{1}{16}\right) = \frac{14}{16}$

$P(A \cap B) = \frac{4}{16} \neq P(A) \cdot P(B)$, not independent.
\[ P(A) = \frac{2}{6} = \frac{1}{3} \]

\[ P(B) = \frac{6+5+4+3+2+1}{36} = \frac{21}{36} = \frac{7}{12} \]

A \cap B occurs for (2, 3), (2, 4), (2, 5), (2, 6), (3, 4), (3, 5), (3, 6), (4, 5), (4, 6), (5, 6)

\[ P(A \cap B) = \frac{7}{36} \neq P(A) P(B) \]

\[ \text{Independent} \]

23 \[ P(A) = \frac{1}{2} \quad P(B) = \frac{1}{2} \quad P(A \cap B) = \frac{9}{36} = \frac{1}{4} \]

\[ A, B \ \text{indep.} \]

\[ P(C) = \frac{1}{2} \quad \text{since red is odd and green is even} \]

\[ P(A \cap C) = \frac{1}{4} = P(A) P(C) \quad \text{indep} \]

\[ P(B \cap C) = \frac{1}{4} = P(B) P(C) \quad \text{indep} \]

\[ A \cap B \cap C \text{ never occurs so} \]

\[ P(A \cap B \cap C) = 0 \neq P(A) P(B) P(C) \]

The three are not independent.
Seat Al in 7 ways, Bob in 8 for 72 total. 18 ways in same row.

\[ P(A) = \frac{18}{72} = \frac{1}{4}. \]

If Al is in a corner, there are 4 choices for Al and the 3 for Bob so

\[ P(B) = \frac{12}{72} = \frac{1}{6}. \]

\[ P(\text{A and B}) = \frac{4}{72} = \frac{1}{18} \neq P(A)P(B). \]

\[ \text{not independent.} \]

25. \[ P(\text{All in about}) = \frac{1}{5} \]
\[ P(\text{Betty about}) = \frac{2}{5}. \]
\[ P(\text{Bert about}) = \frac{2}{25}. \]

\[ P(\text{all least one attends}) = 1 - \frac{2}{25} = \frac{23}{25}. \]

(b) \[ P(\text{exactly one or both}) = \frac{23}{25} - \frac{4}{5} \frac{3}{5} = \frac{11}{25}. \]
Each fails to solve with prob. \( 2/3 \).

All three fail with prob \( (2/3)^3 = 8/27 \).

At least one solves, with prob. \( 1 - 8/27 = 19/27 \).

The expected prize is

\[
100 \left( \frac{1}{100} \right) + 25 \frac{2}{100} + 10 \frac{5}{100} = 2.
\]

Pay up to $2

Think of this as \((\#1) + (\#2 + \#3)\).

The values and prob. for 2 die are

\[
\begin{array}{cccccccc}
1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 \\
(1 & 2 & 3 & 4 & 5 & 6 & 5 & 4 & 3 & 2 & 1 & )/36
\end{array}
\]

11 occurs as \(1+10, 2+9, 3+8, 4+7, 5+6, 6+5\)

for a total of \(3+4+5+6+5+4 = 27\) ways.

12

\(1+11, 2+10, \ldots, 6+6\) in

\(2 + 3 + 4 + 5 + 6 + 5 = 25\) ways, but we
don't allow \(4+4+4\) in 24 ways.

\(1+12, 2+11, \ldots, 6+7\) in

\(1+2+3+4+5+6 = 21\) ways.

\(2+12, 3+11, \ldots, 6+8\) in \(1+2+3+4+5 = 15\) ways.

\(3+12, 4+11, \ldots, 6+9\) in \(1+2+3+4 = 10\) ways (but \(5+5+5\) is

not allowed so 9 ways.)
16: \( 4 + 12 \), \( 5 + 11 \), \( 6 + 12 \) \( \Rightarrow \) 
\[ 1 + 2 + 3 = 6 \text{ ways} \]

17: \( 5 + 12 \), \( 6 + 11 \) \( \Rightarrow \) 
\[ 1 + 2 = 3 \text{ ways} \]

**Total**: \( 27 + 24 + 21 + 15 + 9 + 6 + 3 = 105 \text{ ways} \).

\[ \text{Prob winning} = \frac{105}{6^6} = \frac{105}{216} \]

\[ \mathbb{E} = 1 \cdot \frac{105}{216} - 1 \left( \frac{11}{216} \right) = \frac{6}{216} = \frac{1}{36} \]

47. **Success = "penny is selected"**

\[ P(1 \text{ head}) + P(1 \text{ tail}) = \frac{5}{32} + \frac{5}{32} = \frac{5}{16} \]

\[ P(\text{success}) = \frac{5}{16} . \]

**Expected value**: \[ \frac{1}{5/16} = \frac{16}{5} = 3.2 \text{ rounds} . \]

48. **Success at the \( n \text{th} \) try is \( B \cdots B \cdots G \text{ or } G \cdots G \cdots B \)**

So \[ P(N = n) = \frac{1}{2^n} + \frac{1}{2^n} = \frac{1}{2^{n-1}} , \text{ for } n \geq 2 \]

\[ E(N) = \sum_{n=2}^{\infty} \frac{n}{2^{n-1}} . \]

\[ 1 + x + x^2 + \cdots = \left( 1 - x \right)^{-1} , \text{ for } |x| < 1 \]

\[ 1 + 2x + 3x^2 + \cdots = \left( 1 - x \right)^{-2} , \text{ for } |x| < 1 \]

\[ 2x + 3x^2 + \cdots = \left( 1 - x \right)^{-2} - 1 , \text{ for } |x| < 1 \]

\[ P(\text{and } x = \frac{1}{2}) \]

\[ \frac{1}{2^2} + \frac{3}{2^3} + \frac{4}{2^3} = 4 - 1 = 3 \Rightarrow E(N) = 3 \]