Solutions #4

27 (a) Prob. of success = \( \frac{1}{2} \)

\[ \text{Prob (78 successes)} = \binom{10}{8} \left( \frac{1}{2} \right)^8 \left( \frac{1}{2} \right)^{10} + \binom{10}{9} \left( \frac{1}{2} \right)^9 \left( \frac{1}{2} \right)^{10} + \binom{10}{10} \left( \frac{1}{2} \right)^{10} \]

(b) Prob of success = \( \frac{9}{10} \)

\[ \text{Prob (78 successes)} = \binom{10}{8} \left( \frac{9}{10} \right)^8 \left( \frac{1}{10} \right)^2 + \binom{10}{9} \left( \frac{9}{10} \right)^9 \left( \frac{1}{10} \right)^1 + \binom{10}{10} \left( \frac{9}{10} \right)^{10} \]

40 Since Poisson (a) distribution has expected value \( \lambda \), we take \( \lambda = 2 \). Prob (less than 5 calls)

\[ = \left( 1 + 2 + \frac{2^2}{2!} + \frac{2^3}{3!} + \frac{2^4}{4!} \right) e^{-2} \]

44 \( \lambda = (0.002)(1500) = 3 \)

\[ P(0 \text{ defects}) = e^{-3} \]

47 \( \lambda = \frac{3000}{1000} = 3 \)

\[ P(\leq 2 \text{ accidents}) = \left( 1 + 3 + \frac{3^2}{2!} \right) e^{-3} \]
There are 13 spades and 39 non-spades.

There are \(C_{52,5}\) possible hands. There are \(C_{13,2} \cdot C_{39,3}\) hands with exactly 2 spades.

\[
\text{Prob} = \frac{C_{13,2} \cdot C_{39,3}}{C_{52,5}}
\]

There are \(C_{100,7}\) possibilities.

To get 5 aces and 2 aces: \(C_{54,5} \cdot C_{44,2}\)

\[
\text{Prob} = \frac{C_{54,5} \cdot C_{44,2}}{C_{100,7}}
\]

There are \(C_{16,5}\) choices.

(a) no pair. 16 choices for #1

14 - - - - #2
12 - - - - #3
10 - - - - #4
8 - - - - #5

Each occurs \(5!\) times so

\[
P(\text{no pair}) = \frac{16 \cdot 14 \cdot 12 \cdot 10 \cdot 8}{5! \cdot C_{16,5}}
\]
(b) choose a pair : 8 choices

14 choice 8 w # 3
12 - - - - # 4 \{ 3! repeats. \}
10 - - - - # 5

\[ \text{Prob} = \frac{8 \cdot 14 \cdot 12 \cdot 10}{3! \cdot \binom{16}{5}} \]

(c) choose 2 pairs \( \binom{8}{2} \)

Choose 1 of the remaining 12.

\[ \text{Prob} = \frac{12 \binom{8}{2}}{\binom{16}{5}} \]

65 There are 32 cards \( \leq 9 \).

\[ \text{Prob (All are } \leq 9) = \frac{\binom{32}{13}}{\binom{52}{13}} \]

67 The prob of nothing 6 is \( \frac{C_{9,6}}{C_{12,6}} \)

\[ \text{Prob of nothing 5 is } \frac{C_{9,5} \cdot C_{2,1}}{C_{12,6}} \]

\[ \text{Prob of at least 5 is } \frac{C_{9,6} + C_{9,5} \cdot C_{3,1}}{C_{12,6}} \]
(a) \( \binom{30}{4} \) choices.

To get no two with the same number:

- 30 choices for #1
- 28 choices for #2
- 26 choices for #3
- 24 choices for #4

\[ \text{Ans} = \frac{30 \cdot 28 \cdot 26 \cdot 24}{4! \cdot \binom{30}{4}} \]

(b) Choose the pair: 15 choices.

- 28 choices for ball #3
- 26 choices for ball #4

\[ \text{Prob.} = \frac{15 \cdot 28 \cdot 26}{2 \cdot \binom{30}{4}} \]

(c) Choose 2 pairs: \( \binom{15}{2} \)

\[ \text{Prob.} = \frac{\binom{15}{2}}{\binom{30}{4}} \]