MATH 222, TEST 1

Show all steps for credit. 10 pts. per question

Q1. Find the value of a which makes the set of equations

$$\begin{aligned} x_1 + x_2 + x_3 &= 2 \\ x_1 + 2x_2 + 3x_3 &= 1 \\ 3x_1 + 4x_2 + 5x_3 &= a \end{aligned}$$

consistent, and then find all solutions when a is replaced by this value.

Q2. Find the inverse of the matrix

$$\begin{pmatrix} 1 & 1 & 2 \\ 1 & 2 & 3 \\ 1 & 3 & 5 \end{pmatrix}.$$

Q3. A and C are invertible $n \times n$ matrices. Find X, Y and Z in terms of A, B and C so that

$$\begin{pmatrix} A & 0 \\ B & C \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} X & 0 \\ Y & Z \end{pmatrix}$$

are inverses of one another.

Q4. Write the vector $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$ as a linear combination of the vectors $\begin{pmatrix} 1 \\ 4 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ 5 \end{pmatrix}$.

Q5. Find elementary matrices so that $E_2 E_1 A = I$ where

$$A = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}.$$

Q6. Is $\{x^2+x+1,\ 2x^2+4x+5,\ x^2+2x+3\}$ a spanning set for P_3?

Q7. In the vector space $R^{n\times n}$ of $n\times n$ matrices, let A and B be fixed matrices. Is

$$W=\{X\in R^{n\times n}: AX=XB\}$$

a subspace of $R^{n\times n}$?

Q8. A and B are 3×3 matrices with

$$\det(A)=2, \qquad \det(B)=3.$$

Find the determinants of

$$AB,\ \ A^{-1}B,\ \ 2B,\ \ 3A^{-1},\ \ B^{-1}A^T.$$

Q9. Find the null space of the matrix

$$A=\begin{pmatrix} 1 & 1 & 1 & -3 \\ 1 & 2 & 2 & -5 \end{pmatrix}.$$

Q10. Let $\{\mathbf{v}_1,\ \mathbf{v}_2,\ \mathbf{v}_3\}$ be vectors in a vector space V. Prove that a vector $\mathbf{v}\in \operatorname{span}\{\mathbf{v}_1,\ \mathbf{v}_2,\ \mathbf{v}_3\}$ can be written uniquely as a linear combination of these three vectors if and only if $\{\mathbf{v}_1,\ \mathbf{v}_2,\ \mathbf{v}_3\}$ is a linearly independent set.