SOLUTIONS

MATH 222, QUIZ 3.

NAME________________
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Show all steps for credit.

Q1. (3 pts.) Find all values of λ so that the matrix

$$\begin{pmatrix} 2-\lambda & 4 \\ 3 & 3-\lambda \end{pmatrix}$$

has no inverse.

We want $\begin{vmatrix} 2-\lambda & 4 \\ 3 & 3-\lambda \end{vmatrix} = 0$, so

$(2-\lambda)(3-\lambda) - 12 = 0$, $\lambda^2 - 5\lambda - 6 = 0$,

$(\lambda-6)(\lambda+1) = 0$, $\underline{\lambda = 6 \text{ or } -1}$

Q2. (4 pts.) Find all choices of c which would make the matrix

$$A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 9 & c \\ 1 & c & 3 \end{pmatrix}$$

non–invertible.

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & 9 & c \\ 1 & c & 3 \end{vmatrix} = \begin{vmatrix} 9 & c \\ c & 3 \end{vmatrix} - \begin{vmatrix} 1 & c \\ 1 & 3 \end{vmatrix} + \begin{vmatrix} 1 & 9 \\ 1 & c \end{vmatrix}$$

$= 27 - c^2 - 3 + c + c - 9$

$= 15 + 2c - c^2 = -(c^2 - 2c - 15)$

$= -(c-5)(c+3)$

so $\underline{c = 5 \text{ or } -3}$

Q3. (3 pts.) If the usual addition on $\mathbb{R}^2$ is replaced by

$$(x_1, x_2) \oplus (y_1, y_2) = (x_1 + y_1, 0),$$

show there is no vector that plays the role of the zero vector (i.e. this is not a vector space).

Try (a, b) for the zero vector.

We want

$$(x_1, x_2) \oplus (a, b) = (x_1, x_2).$$

But this is

$$(x_1 + a, 0) = (x_1, x_2)$$

which is impossible for $x_2 \neq 0$.