SOLUTIONS

MATH 222, QUIZ 3.

NAME____________________
ROW____________________

Show all steps for credit.

Q1. (3 pts.) Find all values of $\lambda$ so that the matrix

\[
\begin{pmatrix}
2 - \lambda & 4 \\
3 & 3 - \lambda
\end{pmatrix}
\]

has no inverse.

We want

\[
\begin{vmatrix}
2 - \lambda & 4 \\
3 & 3 - \lambda
\end{vmatrix} = 0,
\]

\[
(2 - \lambda)(3 - \lambda) - 12 = 0,
\]

\[
\lambda^2 - 5\lambda - 6 = 0,
\]

\[
(\lambda - 6)(\lambda + 1) = 0,
\]

$\lambda = 6$ or $-1$.

Q2. (4 pts.) Find all choices of $c$ which would make the matrix

\[
A = \begin{pmatrix}
1 & 1 & 1 \\
1 & 9 & c \\
1 & c & 3
\end{pmatrix}
\]

non-invertible.

\[
\begin{vmatrix}
1 & 1 & 1 \\
9 & c & 1 \\
1 & 3 & 1
\end{vmatrix} = 1 \begin{vmatrix}
9 & c & 1 \\
1 & 3 & 1
\end{vmatrix} + 1 \begin{vmatrix}
1 & 1 & 1 \\
9 & c & 1
\end{vmatrix}
\]

\[
= 27 - c^2 - 3 + c + c - 9
\]

\[
= 15 + 2c - c^2 = -(c^2 - 2c - 15)
\]

\[
= - (c - 5)(c + 3)
\]

$\Rightarrow c = 5$ or $c = -3$.
Q3. (3 pts.) If the usual addition on $\mathbb{R}^2$ is replaced by

$$(x_1, x_2) \oplus (y_1, y_2) = (x_1 + y_1, 0),$$

show there is no vector that plays the role of the zero vector (i.e. this is not a vector space).

Try $(a, b)$ for the zero vector.

We want

$$(x_1, x_2) \oplus (a, b) = (x_1, x_2).$$

But this is

$$(x_1 + a, 0) = (x_1, x_2)$$

which is impossible for $x_2 \neq 0$. 