SOLUTIONS

**MATH 222, QUIZ 4.**

NAME________________
ROW________________

*Show all steps for credit.*

Q1. (3 pts.) Is $\{x+2, x+1, x^2-1\}$ a spanning set for $P_3$?

Try $\alpha_1(x+2)+\alpha_2(x+1)+\alpha_3(x^2-1) = a_0+a_1x+a_2x^2$

$x^2$: $\alpha_3 = a_2$

$x$: $\alpha_1+\alpha_2 = a_1$

$x^0$: $2\alpha_1+\alpha_2-\alpha_3 = a_0$

$$\left(\begin{array}{ccc|c} 1 & 1 & 0 & a_1 \\ 2 & 1 & -1 & a_0 \\ 0 & 0 & 1 & a_2 \end{array}\right) \longrightarrow \left(\begin{array}{ccc|c} 1 & 1 & 0 & a_1 \\ 0 & -1 & -1 & a_0-2a_1 \\ 0 & 0 & 1 & a_2 \end{array}\right)$$

so solutions always exist.

Yes, it is a spanning set.

Q2. (4 pts.) Let $U$ and $W$ be subspaces of a vector space $V$. Prove that the intersection $U \cap W$ is a subspace of $V$.

If $v_1, v_2 \in U\cap W$ then $v_1, v_2 \in U$ and $v_1, v_2 \in W$.

Then $v_1+v_2 \in U$, $v_1+v_2 \in W$, so

$v_1+v_2 \in U\cap W$. Closed under addition.

If $v \in U\cap W$ and $\alpha \in \mathbb{R}$ then

$\alpha v \in U$ and $\alpha v \in W$ so

$\alpha v \in U\cap W$. Closed under scalar multiplication.

Thus $U\cap W$ is a subspace.

Q3. (3 pts.) Is

$$\{(1,1,1)^T,\ (1,3,2)^T,\ (5,9,7)^T\}$$

a linearly independent set of vectors in $\mathbb{R}^3$?

Try $\alpha_1 \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} + \alpha_2 \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix} + \alpha_3 \begin{pmatrix} 5 \\ 9 \\ 7 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

$$\left(\begin{array}{ccc|c} 1 & 1 & 5 & 0 \\ 1 & 3 & 9 & 0 \\ 1 & 2 & 7 & 0 \end{array}\right) \xrightarrow[R_3 - R_1]{R_2 - R_1} \left(\begin{array}{ccc|c} 1 & 1 & 5 & 0 \\ 0 & 2 & 4 & 0 \\ 0 & 1 & 2 & 0 \end{array}\right)$$

$$\xrightarrow[R_2/2]{R_3 - \frac{1}{2}R_2} \left(\begin{array}{ccc|c} 1 & 1 & 5 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

Could choose $\alpha_3 = 1$, $\alpha_2 = -2$ $\alpha_1 = -3$

so they are linearly dependent.