SOLUTIONS

MATH 222, QUIZ 5.

NAME________________________
ROW________________________

Show all steps for credit.

Q1. (3 pts.) Is
\( \{ (1, 1, 1)^T, (1, 2, 2)^T, (1, 1, 2)^T \} \) a basis for \( \mathbb{R}^3 \)?

Try
\[
\alpha_1 \left( \begin{array}{c} 1 \\ 1 \\ 1 \end{array} \right) + \alpha_2 \left( \begin{array}{c} 1 \\ 2 \\ 2 \end{array} \right) + \alpha_3 \left( \begin{array}{c} 1 \\ 1 \\ 2 \end{array} \right) = \left( \begin{array}{c} a \\ b \\ c \end{array} \right)
\]

\[
\left( \begin{array}{ccc} 1 & 1 & 0 \\ 1 & 2 & 0 \\ 1 & 1 & 0 \end{array} \right) \left( \begin{array}{ccc} \alpha_1 & \alpha_2 & \alpha_3 \end{array} \right) \rightarrow \left( \begin{array}{ccc} 1 & 1 & 0 \\ 0 & 0 & b-a \\ 0 & 0 & c+b-a \end{array} \right)
\]

It is spanning and if \( a = b = c = 0 \) then \( \alpha_1 = \alpha_2 = \alpha_3 = 0 \) so li.

Yes it is a basis.

Q2. (4 pts.) Show that \( \{ x + 2, x + 3 \} \) is a basis for \( P_2 \) but not for \( P_3 \).

\( P_2 \)

Try
\[
\alpha_1 (x + 2) + \alpha_2 (x + 3) = a_0 + a_1 x
\]

\[
\begin{align*}
\alpha_1 + \alpha_2 &= a_1 \\
2\alpha_1 + 3\alpha_2 &= a_0
\end{align*}
\]

so we can always solve (spanning) and when \( a_0 = a_1 = 0 \) we get \( \alpha_1 = \alpha_2 = 0 \) (i.e.) yes, a basis for \( P_2 \).

\( P_3 \)

Try
\[
\alpha_1 (x + 2) + \alpha_2 (x + 3) = a_0 + a_1 x + a_2 x^2
\]

\[
\begin{align*}
\alpha_1 + \alpha_2 &= a_1 \\
0\alpha_1 + a_2 &= a_2
\end{align*}
\]

which cannot be solved for \( a_2 \neq 0 \). This is not spanning.

No not a basis.
Q3. (3 pts.) Let \( \{v_1, v_2, \ldots, v_k\} \) be a linearly independent set of vectors in \( \mathbb{R}^n \) and let \( A \) be an invertible \( n \times n \) matrix. Prove that the set of vectors \( \{Av_1, Av_2, \ldots, Av_k\} \) is also a linearly independent set.

If \( \alpha_1 Av_1 + \ldots + \alpha_k Av_k = 0 \)

then multiply by \( A^{-1} \).

This gives

\[ \alpha_1 v_1 + \ldots + \alpha_k v_k = 0. \]

By l.i. of \( \{v_1, \ldots, v_k\} \) we get

\[ \alpha_1 = 0, \alpha_2 = 0, \ldots, \alpha_k = 0. \]

Thus \( \{Av_1, \ldots, Av_k\} \) is l.i.