SOLUTIONS

MATH 222, QUIZ 5. **NAME**________________

ROW________________

Show all steps for credit.

Q1. (3 pts.) Is

$$\{(1,1,1)^T,\ (1,2,2)^T,\ (1,1,2)^T\}$$

a basis for $\mathbb{R}^3$?

Try $\alpha_1\begin{pmatrix}1\\1\\1\end{pmatrix} + \alpha_2\begin{pmatrix}1\\2\\2\end{pmatrix} + \alpha_3\begin{pmatrix}1\\1\\2\end{pmatrix} = \begin{pmatrix}a\\b\\c\end{pmatrix}$

$$\left(\begin{array}{ccc|c}1&1&1&a\\1&2&1&b\\1&2&2&c\end{array}\right) \xrightarrow[R_3-R_1]{R_2-R_1} \left(\begin{array}{ccc|c}1&1&1&a\\0&1&0&b-a\\0&1&1&c\end{array}\right) \xrightarrow{R_3-R_2} \left(\begin{array}{ccc|c}1&1&1&a\\0&1&0&b-a\\0&0&1&c-b+a\end{array}\right).$$

It is spanning and if $a=b=c=0$ then $\alpha_1=\alpha_2=\alpha_3=0$ so l.i.

Yes it is a basis

Q2. (4 pts.) Show that $\{x+2, x+3\}$ is a basis for P_2 but not for P_3.

$\underline{\underline{P_2}}$ Try $\alpha_1(x+2)+\alpha_2(x+3) = a_0 + a_1 x$

$\underline{x}]$ $\alpha_1 + \alpha_2 = a_1$. $\qquad \left(\begin{array}{cc|c}1&1&a_1\\2&3&a_0\end{array}\right) \xrightarrow{R_2-2R_1} \left(\begin{array}{cc|c}1&1&a_1\\0&1&a_0-2a_1\end{array}\right)$

$x^0]$ $2\alpha_1 + 3\alpha_2 = a_0$.

so we can always solve (spanning) and when $a_0 = a_1 = 0$ we get $\alpha_1=\alpha_2=0$ (l.i.) $\underline{\text{Yes}}$, a basis for P_2.

$\underline{\underline{P_3}}$ Try $\alpha_1(x+2) + \alpha_2(x+3) = a_0 + a_1 x + a_2 x^2$

$\underline{x^2}]$ $0\alpha_1 + 0\alpha_2 = a_2$ which cannot be

solved for $a_2 \neq 0$. This is not spanning

so $\underline{\text{not}}$ a basis.

Q3. (3 pts.) Let $\{v_1, v_2, \ldots, v_k\}$ be a linearly independent set of vectors in $\mathbb{R}^n$ and let A be an invertible $n \times n$ matrix. Prove that the set of vectors $\{Av_1, Av_2, \ldots, Av_k\}$ is also a linearly independent set.

If $\alpha_1 A v_1 + \ldots + \alpha_k A v_k = 0$

then multiply by A^{-1}.

This gives

$$\alpha_1 v_1 + \ldots + \alpha_k v_k = 0.$$

By l.i. of $\{v_1, \ldots, v_k\}$ we get

$$\alpha_1 = 0, \alpha_2 = 0, \ldots, \alpha_k = 0.$$

Thus $\{Av_1, \ldots, Av_k\}$ is l.i.