Solutions

MATH 222, QUIZ 8.

NAME___________________
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Show all steps for credit.

Q1. (5 pts.) a Linear transformation $L : \mathbb{R}^2 \to \mathbb{R}^2$ satisfies

$$L((1,2)^T) = (3,4)^T, \quad L((1,3)^T) = (2,4)^T.$$

Find $L((1,0)^T)$.

$$\begin{pmatrix}1\\0\end{pmatrix} = \alpha_1 \begin{pmatrix}1\\2\end{pmatrix} + \alpha_2 \begin{pmatrix}1\\3\end{pmatrix}$$

$$\left(\begin{array}{cc|c} 1 & 1 & 1 \\ 2 & 3 & 0 \end{array}\right) \to \left(\begin{array}{cc|c} 1 & 1 & 1 \\ 0 & 1 & -2 \end{array}\right)$$

$$\alpha_2 = -2, \ \alpha_1 = 3$$

$$\begin{pmatrix}1\\0\end{pmatrix} = 3\begin{pmatrix}1\\2\end{pmatrix} - 2\begin{pmatrix}1\\3\end{pmatrix}$$

$$L\begin{pmatrix}1\\0\end{pmatrix} = 3\begin{pmatrix}3\\4\end{pmatrix} - 2\begin{pmatrix}2\\4\end{pmatrix}$$

$$= \begin{pmatrix}9\\12\end{pmatrix} - \begin{pmatrix}4\\8\end{pmatrix} = \begin{pmatrix}5\\4\end{pmatrix}.$$

Q2. (5 pts.) Prove that a linear transformation $L : V \to W$ is *one-to-one* if and only if its kernel is $\{0\}$.

If L is one to one and $v \in \ker L$ then

$L(v) = 0 = L(0)$,

so $v = 0$ and $\ker L = \{0\}$.

Conversely, if $\ker L = \{0\}$,

suppose that $L(v_1) = L(v_2)$.

Then $L(v_1 - v_2) = 0$ so

$v_1 - v_2 \in \ker L = \{0\}$.

$\therefore$ $v_1 - v_2 = 0$ so

$\underline{v_1 = v_2}$ and L is one-to-one.