Solutions

MATH 222, QUIZ 9.

NAME________________

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Show all steps for credit.

Q1. (5 pts.) A linear transformation $L : \mathbb{R}^2 \to \mathbb{R}^{2\times 2}$ is defined by

$$L((x_1, x_2)^T) = \begin{pmatrix} x_2 & x_2 \\ x_1 & x_2 \end{pmatrix}.$$

Find the matrix of L when the first space has the standard basis and the second has the basis

$$\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}.$$

$$L\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$$

$$L\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} + \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

so the matrix is

$$\cancel{\begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}} \quad \underline{\begin{pmatrix} 0 & 1 \\ 0 & 1 \\ 1 & 0 \\ 0 & 1 \end{pmatrix}}$$

Q2. (5 pts.) If A and B are similar, prove that A^2 and B^2 are similar.

If $A = S^{-1} B S$ then

$$A^2 = (S^{-1} B S)(S^{-1} B S)$$

$$= S^{-1} B^2 S$$

so A^2 and B^2 are similar