SOLUTIONS

MATH 222, QUIZ 11.

NAME________________
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Show all steps for credit.

Q1. (5 pts.) Find an invertible S and a diagonal D so that $S^{-1}AS = D$, where

$$A = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}.$$

$$\begin{vmatrix} 1-\lambda & 1 & 1 \\ 0 & 2-\lambda & 0 \\ 0 & 0 & 3-\lambda \end{vmatrix} = (1-\lambda)(2-\lambda)(3-\lambda) = 0$$

$$\lambda = 1, 2, 3$$

$\underline{\lambda=1}$ $\left(\begin{array}{ccc|c} 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \end{array}\right) \Rightarrow x_2 = 0, x_3 = 0$. Take $\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$

$\underline{\lambda=2}$ $\left(\begin{array}{ccc|c} -1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right)$ Take $\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$

$\underline{\lambda=3}$ $\left(\begin{array}{ccc|c} -2 & 1 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$ Take $\begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}$

$$\underline{S = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix}}, \quad \underline{D = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}}$$

Q2. (5 pts.) Diagonalize

$$A = \begin{pmatrix} 7 & -3 \\ 6 & -2 \end{pmatrix},$$

and then find B so that $B^2 = A$.

$$\begin{vmatrix} 7-\lambda & -3 \\ 6 & -2-\lambda \end{vmatrix} = \lambda^2 - 5\lambda + 4 = (\lambda-1)(\lambda-4) = 0$$

$$\lambda = 1, 4$$

$\underline{\lambda = 1}$ $\quad \left(\begin{array}{cc|c} 6 & -3 & 0 \\ 6 & -3 & 0 \end{array}\right) \to \left(\begin{array}{cc|c} 2 & -1 & 0 \\ 0 & 0 & 0 \end{array}\right)$ Take $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$

$\underline{\lambda = 4}$ $\quad \left(\begin{array}{cc|c} 3 & -3 & 0 \\ 6 & -6 & 0 \end{array}\right) \to \left(\begin{array}{cc|c} 1 & -1 & 0 \\ 0 & 0 & 0 \end{array}\right)$ Take $\begin{pmatrix} +1 \\ 1 \end{pmatrix}$

$$S = \begin{pmatrix} 1 & +1 \\ 2 & 1 \end{pmatrix}, \quad D = \begin{pmatrix} 1 & 0 \\ 0 & 4 \end{pmatrix}, \quad \sqrt{D} = \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}$$

$$\sqrt{A} = S\sqrt{D}\,S^{-1} = \begin{pmatrix} 1 & +1 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix} \begin{pmatrix} -1 & 1 \\ 2 & -1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 2 \\ 2 & 2 \end{pmatrix} \begin{pmatrix} -1 & 1 \\ 2 & -1 \end{pmatrix} = \begin{pmatrix} 3 & -1 \\ 2 & 0 \end{pmatrix}$$