MATH 447, HOMEWORK 1, DUE THURSDAY JAN 24th

Q1. Let $f(x)$ be a bounded function on $[a,b]$. Suppose that there is a sequence of partitions P_n so that

$$\lim_{n\to\infty} (\overline{S}(f,P_n) - \underline{S}(f,P_n)) = 0.$$

Prove that the upper and lower integrals are the same, and that

$$\int_a^b f(x)\,dx = \lim_{n\to\infty} \overline{S}(f,P_n).$$

Q2. Use Q1 to evaluate $\int_0^1 x\,dx$ by upper and lower sums.

Q3. Let (X,ρ) be a compact metric space and let $\{f_n\}_{n=1}^{\infty}$ be a sequence of continuous functions converging pointwise to a continuous function f. If $f_n(x) \le f_{n+1}(x)$ for all $x \in X$ and $n \ge 1$, then prove that the convergence is uniform.

Q4. Prove that $\sum_{n=0}^{\infty} x^n$ converges pointwise but not uniformly to $(1-x)^{-1}$ on $[0,1)$. Prove that convergence is uniform on any compact subinterval.

Q5. Let $f_n(x) = nxe^{-n^2x^2}$, $n \ge 1$. Prove that $f_n \to 0$ pointwise but not uniformly on $[0,1]$.