

MATH 447, HOMEWORK 3, DUE THURSDAY FEB 7th

Q1. On $[0, 1] \times [0, 1]$, let

$$A = \text{span}\{f(x)g(y) : f, g \in C[0, 1]\}.$$

Prove that $\overline{A} = C([0, 1] \times [0, 1])$.

Q2. Let

$$A = \text{span}\{1, x^2, x^4, x^6, \dots\}$$

on $[-1, 1]$. Prove that \overline{A} is the set of even continuous functions on this interval.

Q3. On $[-\pi, \pi]$, let

$$A = \text{span}\{\sin nx, \cos nx : n \geq 0\}.$$

Prove that $\overline{A} = \{f \in C[-\pi, \pi] : f(-\pi) = f(\pi)\}$.

Q4. Prove that any continuous function f on $[0, \pi]$ satisfying $f(0) = f(\pi) = 0$ extends to an odd continuous function on $[-\pi, \pi]$. Use Q3 to prove that f is in the uniform closure of $\text{span}\{\sin nx : n \geq 1\}$ on $[0, \pi]$.

Q5. Let X be a compact metric space and let A be an algebra of continuous functions separating points. Prove that \overline{A} is either $C(X)$ or $\{f \in C(X) : f(x_0) = 0\}$ for some fixed point $x_0 \in X$.