MATH 447, HOMEWORK 7, DUE Mar 27th

Q1. Let $\chi_E$ be the characteristic function of a set $E$. Prove that $\chi_E$ is measurable if and only if $E \in \mathcal{M}$.

Q2. Let $f : \mathbb{R} \to \mathbb{R}$ be measurable. Prove that
\[
\{ F \subseteq \mathbb{R} : f^{-1}(F) \in \mathcal{M} \}
\]
is a $\sigma$–algebra, and then prove that $f^{-1}(B) \in \mathcal{M}$ for all Borel sets $B$.

Q3. Prove that the pointwise limit of a sequence of measurable functions is measurable.

Q4. Let $f$ be measurable and let $g$ be continuous. Prove that $g(f(x))$ is measurable.

Q5. If $f$ is measurable and $g$ is a function such that $f(x) = g(x)$ everywhere outside a null set, prove that $g$ is measurable.