MATH 447, HOMEWORK 8, DUE Apr 10th

Q1. If \( f \) is continuous on \([0, 1]\) prove that the Riemann and Lebesgue integrals give the same value.

Q2. If \( \int_0^1 f = 0 \) and \( f \geq 0 \), prove that \( f = 0 \) a.e.

Q3. Let \( f_n \) be a sequence of measurable functions on \([0, 1]\) converging pointwise to \( f \). Given \( \varepsilon, \delta > 0 \), prove there is a measurable set \( E \) with \( \mu(E) < \delta \) and an integer \( N \) so that

\[
|f(x) - f_n(x)| < \varepsilon, \quad x \in E^c, \quad n \geq N.
\]

Q4. Let \( f_n \) be a sequence of measurable functions on \([0, 1]\) converging pointwise to \( f \). Given \( \eta > 0 \) prove that there is a measurable set \( E \) with \( \mu(E) < \eta \) so that \( f_n \to f \) uniformly on \( E^c \). (Use Q3)

Q5. If \( f : \mathbb{R} \to \mathbb{R} \) is measurable, prove that \( f(x^2) \) is measurable.