MATH 447, HOMEWORK 9, DUE APR 17

Q1. Let μ be Lebesgue measure, and let $f \geq 0$ be a measurable function. Prove that

$$\nu(E) = \int_E f \, d\mu = \int f \chi_E \, d\mu, \quad E \in \mathcal{M},$$

defines a measure on $\mathbb{R}$.

Q2. Let $f(x) = \sum x^n/n$, $0 \leq x \leq 1$. Prove that $f \in L^1[0,1]$.

Q3. Prove that continuous functions of compact support are dense in $L^1(\mathbb{R})$ in the metric coming from $\| \cdot \|_1$.

Q4. If $f \in L^1(\mathbb{R})$, prove that $\lim_{n\to\infty} \int f(x) \sin^n(x) \, dx = 0$.

Q5. Let $f \in L^1(\mathbb{R})$. Prove that

$$\lim_{n\to\infty} \int f(x) \sin(nx) \, dx = 0.$$

(do it first for step functions)