# MATH 407, FINAL, FALL 2006

*Show all steps for credit. Each question is worth 10 pts.*

Q1. In the region $1 < |z| < 2$, find the Laurent series for

$$\frac{z-2}{(z+1)(z+2)}.$$

Q2. Let $k > 0$ be a constant. Evaluate

$$\int_0^\infty \frac{\log x}{x^2+k^2}\,dx.$$

Q3. Evaluate

$$\int_{|z|=1} \frac{\cosh z}{z^4}\,dz\,.$$

Q4. Let $\gamma$ be any curve joining $3+4i$ to $3-4i$ without crossing the non-negative $x$-axis. Evaluate

$$\int_\gamma \frac{1}{z}\,dz\,.$$

If you use a Logarithm, carefully explain which branch you are using.

Q5. Let

$$f(z) = \begin{cases} \frac{(\overline{z})^2}{z} & \text{when} \quad z \neq 0, \\ 0 & \text{when} \quad z = 0. \end{cases}$$

Show that the Cauchy-Riemann equations are satisfied at the origin $z = 0$ and nowhere else.

Q6. Evaluate

$$\int_0^{2\pi} \frac{1}{5+4\cos\theta}\,d\theta.$$

Q7. Evaluate

$$\int_{|z|=2} \frac{\sin(\pi z)}{z(z-1)^2}\,dz\,.$$

Q8. Evaluate

$$\int_{|z|=1} \frac{e^z}{\sin z}\,dz.$$

Q9.

(i) State Cauchy's integral formulas for $f^{(n)}(0)$ as an integral around $|z| = R$.

(ii) Let $f$ be an entire function satisfying $|f(z)| \leq |z|$. By letting $R \to \infty$ in (i), show that $f^{(n)}(0) = 0$ for $n \geq 2$.

Q10. Evaluate

$$\int_{|z|=1} \frac{(z+1)e^{1/z}}{z^3}\, dz.$$