Problems I didn't discuss in class

Q1
$$\frac{z-2}{(z+1)(z+2)} = \frac{-3}{z+1} + \frac{4}{z+2} = \frac{-3}{z(1+\frac{1}{z})} + \frac{2}{(1+\frac{z}{2})}$$

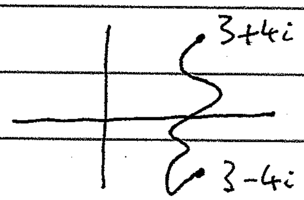
$$= \frac{-3}{z}\left(1 - \frac{1}{z} + \frac{1}{z^2} \cdots\right) + 2\left(1 - \frac{z}{2} + \frac{z^2}{4} \cdots\right)$$

Q3. Use $f^{(3)}(0) = \frac{3!}{2\pi i}\int_{|z|=1} \frac{f(z)}{z^4}\,dz$ with $f(z) = \cosh z$

$f' = \sinh z \quad f'' = \cosh z \quad f''' = \sinh z$ so $f^{(3)}(0) = 0$.

$$\therefore \int_{|z|=1} \frac{\cosh z}{z^4}\,dz = 0.$$

Q4.



Use the branch of Log z

$\mathrm{Log}\, z = \ln|z| + i\,\mathrm{Arg}\, z, \quad -\pi < \mathrm{Arg}\, z < \pi.$

Then
$$\int_{3+4i}^{3-4i} \frac{1}{z}\,dz = \mathrm{Log}(3-4i) - \mathrm{Log}(3+4i)$$
$$= \ln 5 - \tan^{-1}\tfrac{4}{3} - \ln 5 - \tan^{-1}\tfrac{4}{3}$$
$$= 2\tan^{-1}\tfrac{4}{3}.$$

Q7. $\frac{\sin \pi z}{z}$ is analytic so the only pole is at 1.

In powers of $z-1$, $\sin \pi z = \sin \pi + \cos \pi\,(z-1) + \ldots$

and $\frac{1}{z} = \frac{1}{1+(z-1)} = 1 - (z-1) \ldots$

so $\frac{\sin \pi z}{z(z-1)^2} = \frac{(-(z-1) + \ldots)(1-(z-1)\ldots)}{(z-1)^2} = \frac{-1}{z-1} + \ldots$

So $\mathrm{Res}(-, 1) = -1$ and the integral is $\underline{-2\pi i}$.

Q8 Simple pole at 0.

$$\mathrm{Res}\left(\frac{\quad}{\quad}, 0\right) = \lim_{z \to 0} \frac{z e^z}{\sin z} = e^0 = 1$$

so $$\int_{|z|=1} \frac{e^z}{\sin z}\, dz = \underline{2\pi i}.$$