MATH 407, FIRST TEST, FALL 2006

Q1-4, 12 pts each; Q5-8 13 pts each

Q1. (6 pts each)

(i) Put $-1+i$ in polar form.

(ii) Put $3e^{i\pi/6}$ in Cartesian form.

Q2. Find all solutions of the equation $\sin z = 2$.

Q3. Determine all points where $f(z) = (\overline{z})^2$ has a derivative.

Q4. If $\operatorname{Log} z$ is the principal branch of the logarithm ($-\pi < \operatorname{Arg} z < \pi$), determine the quadrants in which the equation

$$\operatorname{Log}(z^2) = 2\operatorname{Log} z$$

is true.

Q5. Evaluate

$$\int_\gamma \overline{z}\, dz,$$

where γ is the straight line path from 0 to $1+i$.

Q6. Find

$$\int_\gamma \frac{1}{z}\, dz,$$

where γ is the straight line path from $-1-i$ to i.

Q7. $f(z)$ is analytic in the plane, $f(0) = 0$, and the real part is

$$x^2 + 2x - y^2 - 3y.$$

Find the imaginary part and then check that the Cauchy-Riemann equations are satisfied.

Q8. Let $f(z)$ be differentiable at $z = i$. Use the definition of the derivative to show that

$$g(z) = \overline{f(\overline{z})}$$

is differentiable at $z = -i$ and that $g'(-i) = \overline{f'(i)}$.