MATH 407, SECOND TEST, FALL 2006

Q1-4, 12 pts each; Q5-8 13 pts each

Q1. Find

$$\int_{|z|=1} \frac{\cos z}{z^4}\, dz.$$

Q2. Find

$$\int_{|z|=2} \frac{1}{z(z-1)^2}\, dz.$$

Q3. Let

$$f(z) = \frac{e^{z^2} - \cos z}{z}, \qquad z \neq 0.$$

Find a value for $f(0)$ which makes this function analytic everywhere, and find the Taylor series for it in powers of z.

Q4. Let

$$f(z) = \frac{1}{z} - \frac{1}{\sin z} = \frac{\sin z - z}{z \sin z}, \quad 0 < |z| < \pi.$$

Find a value for $f(0)$ which makes the function analytic in $|z| < \pi$, and use f to evaluate

$$\int_{|z|=1} \frac{1}{\sin z}\, dz.$$

Q5. Find

$$\int_{|z|=3} \frac{1}{z(z-\pi)^4}\, dz.$$

Q6.

(i) State the maximum modulus theorem.

(ii) Find the maximum value of $|e^z|$ over the square with corners at $1 \pm i$ and $-1 \pm i$.

Q7.

(i) State Morera's theorem.

(ii) Show that

$$\int_0^1 \sqrt{t} e^{tz}\, dt$$

defines an analytic function of z.

Q8. Starting from

$$\frac{1}{1-z} = 1 + z^2 + z^3 + \ldots, \quad |z| < 1,$$

find the Taylor series for

$$\frac{1}{(1+z^2)^2}$$

in powers of z.