MATH 447, HOMEWORK 3, DUE FEB 17
Everyone does Q1-Q5, honors students also do Q6, Q7

Q1. Prove that a function \( f \) between topological spaces is continuous if and only if \( f^{-1}(E) \) is closed for all closed \( E \).

Q2. Let \( f_i: (X_i, T_i) \to (Y_i, S_i), i = 1, 2 \) be continuous functions. Define \( g: (X_1 \times X_2, T_1 \times T_2) \to (Y_1 \times Y_2, S_1 \times S_2) \) by \( g(x_1, x_2) = (f_1(x_1), f_2(x_2)) \). Prove that \( g \) is continuous.

Q3. Let \((X, T)\) be a Hausdorff topological space. Let \( E \) be a compact subset. Prove that \( E \) is closed.

Q4. Let \((X, T)\) be a compact topological space. Let \( E \) be a closed subset. Prove that \( E \) is compact.

Q5. Let \((X, T)\) and \((Y, S)\) be topological spaces. Prove that a net \( ((x_\alpha, y_\alpha)) \) converges to \((x, y)\) in the product topology if and only if \( x_\alpha \to x \) and \( y_\alpha \to y \).

Q6. Let \((X, T)\) and \((Y, S)\) be compact Hausdorff topological spaces. If \( f: X \to Y \) is one-to-one onto and continuous, prove that the inverse function is continuous.

Q7. Let \((X, T)\) be a Hausdorff topological space. \( X \) is said to be locally compact if, given \( x \in X \) there exists an open set \( U \) containing \( x \) and \( \overline{U} \) is compact (for example, the real line). Adjoin a point \( \omega \) to \( X \) to form \( \hat{X} = X \cup \{\omega\} \) and declare a subset to be open if it is an open subset of \( X \) or if it is the complement of a compact subset of \( X \). Prove that \( \hat{X} \) is a compact Hausdorff space, that the identity is a continuous embedding of \( X \) into \( \hat{X} \), and that the real valued continuous functions on \( X \) separate points.