Q1. Let \((X, \Sigma)\) be a measurable space and let \(f, g : X \rightarrow \mathbb{R}\) be measurable functions. Prove that \(\max\{f, g\}\) and \(\min\{f, g\}\) are measurable.

Q2. Let \(f : (X, \Sigma) \rightarrow \mathbb{R}\) be measurable. Prove that \(f^{-1}(B) \in \Sigma\) for all Borel sets \(B\).

Q3. Let \(f : (X, \Sigma) \rightarrow \mathbb{R}\) be measurable and let \(g : \mathbb{R} \rightarrow \mathbb{R}\) be continuous. Prove that \(g \circ f\) is measurable.

Q4. Let \(f : [a, b] \rightarrow [c, d]\) be one-to-one onto and increasing. Prove that \(f^{-1}\) is continuous and increasing.

Q5. Let \(C\) be the Cantor set, let \(f\) be the Cantor function, and let \(g(x) = f(x) + x\). Prove that \(g\) maps \(C\) to a closed set \(D\) of measure 1 in \([0, 2]\). Let \(P\) be a non-Lebesgue set in \(D\). Prove that \(g^{-1}(P)\) is a Lebesgue but non-Borel set in \(C\).

Q6. Let \(\{f_n\}\) be a sequence of measurable functions on \((X, \Sigma)\). Let

\[E = \{x : \{f_n(x)\} \text{ is Cauchy}\}.
\]

Prove that \(E \in \Sigma\).

Q7. Using the results of previous problems prove that there is a continuous function \(f\) and a Lebesgue measurable function \(g\) so that \(g \circ f\) is not Lebesgue measurable.