

SPRING SEMESTER -- 2007: T&R 9:35 –10:50
MATHEMATICS 663 – SEMINAR IN ANALYSIS
OPERATOR THEORY AND COMPLEX GEOMETRY

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The aspect of operator theory I find most fascinating is that Hilbert space operators exist in many different mathematical contexts and their study is amenable to techniques from many areas, not just functional analysis. In the study of a single operator, it is common to borrow from real analysis, complex variables and general topology to name just a few areas. When one ventures into the study of multivariate operator theory, other more sophisticated areas come into play. By now we are familiar with the use of techniques from algebraic topology, homological algebra and differential geometry in the study of self-adjoint operator algebras and the establishment of a field now called “non commutative geometry.” In this course, we will be proceeding in a different direction.

The basic object of study is the notion of a Hilbert module, which is a Hilbert space which is also a module over the polynomials in n variables or some other natural algebra of functions over a domain in complex n -space. In case the domain is the unit disk, the theory generalizes the theory of operators on the Hardy and Bergman spaces as well as the model study of Sz.-Nagy and Foias. Emphasis in the course will be on modules occurring naturally on domains such as the ball and polydisk. In studying submodules one encounters a rigidity phenomenon whose proof depends on techniques from commutative algebra and algebraic geometry. Much of the course will be devoted to the study of the corresponding quotient modules and will utilize notions and techniques from complex geometry, which will be developed.

Prerequisites for the course include a standard first-year graduate background in algebra, topology and geometry as well as a reasonable grounding in functional analysis and operator theory. Other needed material will be developed in the course as we proceed. While there will be no text, one can consult the books “Hilbert modules over functions algebras” by Douglas and Paulsen and “Analytic Hilbert modules” by Chen and Guo. We will also rely on several recent research papers by Douglas and Misra.

There will be no homework or formal assignments in the course although exercises will be given which will test ones understanding of the material covered.