1. Evaluate \[ \int_1^e \frac{(\ln x)^{1/3}}{x} \, dx. \]

2. Compute the area enclosed by the parabola \( y^2 = 3x + 4 \) and the straight line \( y = x \).

3. Find \[ \int_4^9 (\sqrt{x} + \frac{1}{\sqrt{x}})^2 \, dx. \]
4. Find \[ \int_{1}^{2} \frac{2x}{2x+4} \, dx. \]

5. Suppose it is given \( f(5) = 4, f'(5) = 3, f(1) = 5, \) and \( f'(1) = -2. \) Compute \( \int_{1}^{5} x f''(x) \, dx. \)

6. Find the volume generated by rotating the region bounded by \( y = \frac{1}{\sqrt{x+1}}, \ y = 0, \ x = 0, \) and \( x = 1 \) about the \( x \)-axis.
7. If \( g(x) = \int_{\cos(x)}^{5x} \cos(t^2) \, dt \), then find \( g'(x) \).

8. Find the integral \( \int x^5 \, e^{x^2} \, dx \).

9. Let \( f \) be a continuous function satisfying that its average value on the interval \([0, t]\) is equal to \( \tan(t) + t \). Find the function \( f \).
10. Find the area bounded by the curves $x + y^2 = 2$ and $x + y = 0$.

11. Find the volume of the described solid $S$. The base of $S$ is the triangular region with vertices $(0, 0), (2, 0), \text{ and } (0, 1)$. Cross-sections perpendicular to the $x$-axis are equilateral triangles.

12. A tank, of semispherical shape with radius equal to $5\text{ft}$, is full of water.

Find the work required to pump the water out of the outlet. Use the fact that the water weighs $62.5 \text{ lb/ft}^3$. 
13. Let $R$ denote the region enclosed by the x-axis, the line $x = 1$, and the curve $y = \sqrt{x}$. Use the method of cylindrical shells to calculate the volume of the solid obtained by rotating the region $R$ about the line $x = 1$.

14. Find the indefinite integral \( \int \frac{\sqrt{x^2 - 4}}{x} \, dx \).

15. For which values of the fixed positive number $p$ does the improper integral \( \int_0^e \frac{dx}{x(lnx)^p} \) converge?
16. Given the sequence \( \{a_n\} = \{\ln(7 + 5n^2) - \ln(3n^2 + 5)\} \). Compute \( \lim_{n \to \infty} a_n \).

17. Find \( \int x \ln(x) \, dx \).

18. Find the integral \( \int \frac{x^2}{x + 3} \, dx \).
19. The recursive sequence defined by $a_1 = 2$ and $a_{n+1} = 5 - \frac{4}{a_n}$ for $n \geq 1$ converges. Find the limit.

20. Determine if the integral $\int_{1}^{\infty} \frac{1}{\sqrt{x^3 + 1}} \, dx$ converges or not.

21. Find the length of the curve $y = \sqrt{x^3}$ from the point $(0,0)$ to $(4,8)$. 
22. Find the integral \( \int \frac{1}{x^2 - 25} \, dx \) with \( a \neq 0 \).

23. Find the area of the surface obtained by rotating the curve \( y = \sqrt{x} \) with \( 9 \leq x \leq 25 \) about the x-axis.

24. Find the integral \( \int \sin^5(x)\cos^4(x) \, dx \).
25. Find \( \int \frac{x^7}{\sqrt{x^4+1}} \, dx \).

26. Find \( \int \frac{x + 8}{x^3 + 4x} \, dx \).

27. Find \( \int_{1}^{\infty} \frac{\ln(x)}{x^2} \, dx \).
28. Given the curve \( x = cost + tsint \), \( y = sint - tcost \), \( 0 \leq t \leq \frac{\pi}{2} \). SET UP BUT DO NOT EVALUATE an integral to find the area of the surface obtained by rotating the curve about the y-axis.

29. Consider the infinite series \( \sum_{n=1}^{\infty} (-1)^n \frac{n}{n+1} \). Determine if it is convergent or not.

30. Compute the sum of the infinite series \( \sum_{n=1}^{\infty} \frac{1}{(n+2)(n+3)} \).
31. Is the series \( \sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n^{5/2}} \) absolutely convergent?

32. Which is the Maclaurin series expansion of the given function \( f(x) = \cos^2(x^2) \).

33. Find the unit vector in the direction of \( \mathbf{b} - \mathbf{a} \) where \( \mathbf{a} = <0, 2, 1> \) and \( \mathbf{b} = <1, 1, 3> \).
34. Given the triangle with vertices $A(2, -2, 5), B(1, 1, 4),$ and $C(3, 1, 3).$ Find the cosine of the angle at $B.$

35. Find a power representation for the function $f(x) = \frac{x^2}{9 - x^2}$ and determine the interval of convergence.

36. A constant force with vector representation $\mathbf{F} = 10\mathbf{i} + 18\mathbf{j} - 6\mathbf{k}$ moves an object along a straight line from the point $A(2, 3, 0)$ to the point $B(4, 9, 15).$ Find the work done if the distance is measured in meters and the magnitude of the force is measured in Newtons.
37. Find the sum of the series \( \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n-1}}{6^{2n+1}(2n)!} \).

38. Evaluate the integral \( \int \frac{\sin x}{x} \, dx \) as an infinite series.

39. Find the sum of the series \( \sum_{n=0}^{\infty} 5^{-3n+1} \frac{4^{2n+1}}{} \).
40. Consider the following series \( \sum_{n=1}^{\infty} \sin \left( \frac{1}{n^2} \right) \). Determine if the series is convergent.

41. Determine radius and interval of convergence for the series \( \sum_{n=1}^{\infty} (-3)^n \frac{(x + 5)^n}{n} \).

42. Does the series \( \sum_{n=2}^{\infty} (-1)^{n+1} \frac{1}{n \ln^3(n)} \) converge? Does it converge absolutely?
43. Find a series expansion for the function \( f(x) = \frac{1}{(4+x)^2} \).

44. The vertices of a triangle are \( A(2,4,5) \), \( B(3,5,3) \), and \( C(2,8,-3) \). Let \( \mathbf{a} \) be the vector from \( A \) to \( B \), \( \mathbf{b} \) be the vector from \( A \) to \( C \), and \( \mathbf{p} \) the vector projection of \( \mathbf{a} \) onto \( \mathbf{b} \)

(a) Find \( \mathbf{a}, \mathbf{b}, \) and \( \mathbf{p} \)

(b) Find the vector \( \mathbf{a} - \mathbf{p} \) and interpret. What is the distance from the point \( B \) to the line segment joining the points \( A \) and \( C \) ?

45. Using the properties of the cross product, find \( \mathbf{a} \times \mathbf{b} \) if \( \mathbf{a} = \mathbf{i} + \mathbf{j} + \mathbf{k} \) and \( \mathbf{b} = \mathbf{i} + \mathbf{j} - \mathbf{k} \).
46. Find the cosine of the angle between the vectors \( a = i + j + 2k \) and \( b = 2j - 3k \).

47. Evaluate the integral \( \int e^{x^3} \, dx \) as an infinite series.

48. Find the Maclaurin series expansion of the function \( f(x) = \tan^{-1}(x^2) \).
49. Find the sum $\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1} 2^n}{2n + 1}$.

50. Find the sum $\sum_{n=1}^{\infty} \frac{(-1)^n x^{n+2}}{(n + 1)!}$.

51. Given the points $A = (1, 0, 1)$, $B = (1, 1, 1)$ and $C = (1, 6, a)$, determine for what values of $a$ the three points are collinear.