1. Evaluate \( \int_1^e \frac{(\ln x)^{1/3}}{x} \, dx \).

2. Compute the area enclosed by the parabola \( y^2 = 3x + 4 \) and the straight line \( y = x \).

3. Find \( \int_4^9 (\sqrt{x} + \frac{1}{\sqrt{x}})^2 \, dx \).

4. Find \( \int_1^2 \frac{2x}{2x + 4} \, dx \).

5. Suppose it is given \( f(5) = 4, f'(5) = 3, f(1) = 5, \) and \( f'(1) = -2 \). Compute \( \int_1^5 xf''(x) \, dx \).

6. Find the volume generated by rotating the region bounded by \( y = \frac{1}{\sqrt{x^2 + 1}}, y = 0, x = 0, \) and \( x = 1 \) about the \( x \)-axis.

7. If \( g(x) = \int_{\cos(x)}^{5x} \cos(t^2) \, dt \), then find \( g'(x) \).

8. Find the integral \( \int x^5e^x \, dx \).

9. Let \( f \) be a continuous function satisfying that its average value on the interval \([0, t]\) is equal to \( \tan(t) + t \). Find the function \( f \).

10. Find the area bounded by the curves \( x + y^2 = 2 \) and \( x + y = 0 \).

11. Find the volume of the described solid \( S \). The base of \( S \) is the triangular region with vertices \( (0, 0), (2, 0), \) and \( (0, 1) \). Cross-sections perpendicular to the \( x \)-axis are equilateral triangles.

12. A tank, of semispherical shape with radius equal to \( 5 \text{ ft} \), is full of water.

Find the work required to pump the water out of the outlet. Use the fact that the water weighs \( 62.5 \text{ lb/ft}^3 \).

13. Let \( R \) denote the region enclosed by the \( x \)-axis, the line \( x = 1 \), and the curve \( y = \sqrt{x} \). Use the method of cylindrical shells to calculate the volume of the solid obtained by rotating the region \( R \) about the line \( x = 1 \).

14. Find the indefinite integral \( \int \frac{\sqrt{x^2 - 4}}{x} \, dx \).

15. For which values of the fixed positive number \( p \) does the improper integral \( \int_0^e \frac{dx}{x(lnx)^p} \) converge?

16. Given the sequence \( \{a_n\} = \{ln(7 + 5n^2) - ln(3n^2 + 5)\} \). Compute \( \lim_{n \to \infty} a_n \).

17. Find \( \int x\ln(x) \, dx \).

18. Find the integral \( \int \frac{x^2}{x+3} \, dx \).
19. The recursive sequence defined by \( a_1 = 2 \) and \( a_{n+1} = 5 - \frac{4}{a_n} \) for \( n \geq 1 \) converges. Find the limit.

20. Determine if the integral \( \int_1^\infty \frac{1}{\sqrt{x^3 + 1}} \, dx \) converges or not.

21. Find the length of the curve \( y = \sqrt{x^3} \) from the point \((0, 0)\) to \((4, 8)\).

22. Find the integral \( \int \frac{1}{x^2 - 25} \, dx \) with \( a \neq 0 \).

23. Find the area of the surface obtained by rotating the curve \( y = \sqrt{x^3} \) with \( 9 \leq x \leq 25 \) about the x-axis.

24. Find the integral \( \int \sin^5(x)\cos^4(x) \, dx \).

25. Find \( \int \frac{x^7}{\sqrt{x^3 + 1}} \, dx \).

26. Find \( \int \frac{x + 8}{x^3 + 4x} \, dx \).

27. Find \( \int_1^\infty \frac{\ln(x)}{x^2} \, dx \).

28. Given the curve \( x = \cos t + tsin^2, y = \sin t - t\cos^2 \), \( 0 \leq t \leq \frac{\pi}{2} \). SET UP BUT DO NOT EVALUATE an integral to find the area of the surface obtained by rotating the curve about the y-axis.

29. Consider the infinite series \( \sum_{n=1}^{\infty} (-1)^n \frac{n}{n+1} \). Determine if it is convergent or not.

30. Compute the sum of the infinite series \( \sum_{n=1}^{\infty} \frac{1}{(n + 2)(n + 3)} \).

31. Is the series \( \sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n^{3/2}} \) absolutely convergent?

32. Which is the Maclaurin series expansion of the given function \( f(x) = \cos^2(x^2) \).

33. Find the unit vector in the direction of \( \mathbf{b} - \mathbf{a} \) where \( \mathbf{a} = \langle 0, 2, 1 \rangle \) and \( \mathbf{b} = \langle 1, 1, 3 \rangle \).

34. Given the triangle with vertices \( A(2, -2, 5), B(1, 1, 4) \), and \( C(3, 1, 3) \). Find the cosine of the angle at \( B \).

35. Find a power representation for the function \( f(x) = \frac{x^2}{9 - x^2} \) and determine the interval of convergence.

36. A constant force with vector representation \( \mathbf{F} = 10\mathbf{i} + 18\mathbf{j} - 6\mathbf{k} \) moves an object along a straight line from the point \( A(2, 3, 0) \) to the point \( B(4, 9, 15) \). Find the work done if the distance is measured in meters and the magnitude of the force is measured in Newtons.

37. Find the sum of the series \( \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n-1}}{6^{2n+1}(2n)!} \).

38. Evaluate the integral \( \int \frac{\sin x}{x} \, dx \) as an infinite series.

39. Find the sum of the series \( \sum_{n=0}^{\infty} 5^{-3n+1}4^{2n+1} \).

40. Consider the following series \( \sum_{n=1}^{\infty} \sin\left(\frac{1}{n^2}\right) \). Determine if the series is convergent.

41. Determine radius and interval of convergence for the series \( \sum_{n=1}^{\infty} (-3)^n \frac{(x + 5)^n}{n} \).
42. Does the series \( \sum_{n=2}^{\infty} \frac{(-1)^{n+1}}{nln^3(n)} \) converge? Does it converge absolutely?

43. Find a series expansion for the function \( f(x) = \frac{1}{(4 + x)^2} \).

44. The vertices of a triangle are \( A(2, 4, 5) \), \( B(3, 5, 3) \), and \( C(2, 8, -3) \). Let \( \mathbf{a} \) be the vector from \( A \) to \( B \), \( \mathbf{b} \) be the vector from \( A \) to \( C \), and \( \mathbf{p} \) the vector projection of \( \mathbf{a} \) onto \( \mathbf{b} \)

(a) Find \( \mathbf{a} \), \( \mathbf{b} \), and \( \mathbf{p} \)

(b) Find the vector \( \mathbf{a} - \mathbf{p} \) and interpret. What is the distance from the point \( B \) to the line segment joining the points \( A \) and \( C \) ?

45. Using the properties of the cross product, find \( \mathbf{a} \times \mathbf{b} \) if \( \mathbf{a} = i + j + k \) and \( \mathbf{b} = i + j - k \).

46. Find the cosine of the angle between the vectors \( \mathbf{a} = i + j + 2k \) and \( \mathbf{b} = 2j - 3k \).

47. Evaluate the integral \( \int e^{x^2} dx \) as an infinite series.

48. Find the Maclaurin series expansion of the function \( f(x) = \tan^{-1}(x^2) \).

49. Find the sum \( \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{2n + 1} \).

50. Find the sum \( \sum_{n=1}^{\infty} \frac{(-1)^n x^{n+2}}{(n + 1)!} \).

51. Given the points \( A = (1, 0, 1) \), \( B = (1, 1, 1) \) and \( C = (1, 6, a) \), determine for what values of \( a \) the three points are collinear.