1. Determine the sum of the series \[ \sum_{n=1}^{\infty} \left( \frac{1}{\sqrt{n}} - \frac{1}{\sqrt{n+1}} \right). \]

2. Let \( a_n = \frac{2n}{3n+1} \). Determine whether \{a_n\} is convergent. Determine whether \( \sum_{n=1}^{\infty} a_n \) is convergent.

3. Determine whether the series \( 4 + \frac{8}{5} + \frac{16}{25} + \frac{32}{125} + \cdots \) is convergent or divergent. If it is convergent, find its sum.
4. Determine whether the series \( \sum_{n=1}^{\infty} \left( \frac{1}{e^{2n}} \right) \) is convergent or divergent. If it is convergent, find its sum.

5. Determine whether the series \( \sum_{n=1}^{\infty} [2(0.1)^n + (0.2)^n] \) is convergent or divergent. If it is convergent, find its sum.
6. Determine whether the series \[ \sum_{n=1}^{\infty} \frac{n^2}{3(n+1)(n+2)} \] is convergent or divergent. If it is convergent, find its sum.

7. Determine whether the series \[ \sum_{n=1}^{\infty} \frac{1}{n(n+2)} \] is convergent or divergent. If it is convergent, find its sum.

8. Determine whether the series \[ \sum_{n=1}^{\infty} \frac{1}{4n^2 - 1} \] is convergent or divergent. If it is convergent, find its sum.
9. Determine whether the series \( \sum_{n=1}^{\infty} \frac{n}{\sqrt{1 + n^2}} \) is convergent or divergent. If it is convergent, find its sum.

10. Determine whether the series \( \sum_{n=1}^{\infty} \frac{1}{1 + 2^{-n}} \) is convergent or divergent. If it is convergent, find its sum.

11. Find the values for \( x \) such that the series \( \sum_{n=1}^{\infty} (x - 3)^n \) is convergent. Find the sum for those values of \( x \).
12. Find the values for $x$ such that the series $\sum_{n=1}^{\infty} \frac{1}{x^n}$ is convergent. Find the sum for those values of $x$.

13. Find the values for $x$ such that the series $\sum_{n=1}^{\infty} \tan^n x$ is convergent. Find the sum for those values of $x$.

14. Find the sum of the series $\sum_{n=1}^{\infty} \frac{1}{(4n+1)(4n-3)}$. 
15. Find the sum of the series \( \sum_{n=1}^{\infty} \frac{n^2 + 3n + 1}{(n^2 + n)^2} \).

16. If the \( n \)-th partial sum of a series \( \sum_{n=1}^{\infty} a_n \) is 
\[ s_n = \frac{n - 1}{n + 1} \]
find \( a_n \) and \( \sum_{n=1}^{\infty} a_n \).

17. If the \( n \)-th partial sum of a series \( \sum_{n=1}^{\infty} a_n \) is 
\[ s_n = 3 - n2^{-n} \]
find \( a_n \) and \( \sum_{n=1}^{\infty} a_n \).
18. What is the value for $c$ if $\sum_{n=2}^{\infty} (1 + c)^{-n}$ such that the series is convergent?

19. Suppose that $\sum_{n=1}^{\infty} a_n \neq 0$, is known to be convergent. Prove that $\sum_{n=1}^{\infty} \frac{1}{a_n}$ is divergent.

20. Determine whether the following series are absolutely convergent.

(a) $\sum_{n=1}^{\infty} \frac{(-3)^n}{n^3}$.

(b) $\sum_{n=1}^{\infty} \frac{(-3)^n}{n!}$.

(c) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{2n + 1}$. 

(d) \[ \sum_{n=1}^{\infty} \frac{(-1)^{n-1} \sqrt{n}}{n + 1} \].

(e) \[ \sum_{n=1}^{\infty} \frac{(-1)^{n+1} 5^{n+1}}{(n+1)^2 4^{n+2}} \].

(f) \[ \sum_{n=1}^{\infty} \frac{\cos(n\pi/6)}{n \sqrt{n}} \].
21. For which of the following series is the ratio test inconclusive?

(a) \( \sum_{n=1}^{\infty} \frac{1}{n^3} \).

(b) \( \sum_{n=1}^{\infty} \frac{(-3)^{n-1}}{\sqrt{n}} \).

(c) \( \sum_{n=1}^{\infty} \frac{\sqrt{n}}{1 + n^2} \).
22. For which positive integers $k$ is the series

$$\sum_{n=1}^{\infty} \frac{(n!)^2}{(kn)!}$$

convergent?

23. (a) Show that $\sum_{n=0}^{\infty} \frac{x^n}{n!}$ is convergent for all $x$.

(b) Deduce that $\lim_{n \to \infty} \frac{x^n}{n!} = 0$
24. Suppose that the series $\sum_{n=0}^{\infty} c_n x^n$ is convergent when $x = 4$ and diverges when $x = 6$. Is the series $\sum_{n=0}^{\infty} c_n$ convergent? What can be said about the convergence or divergence of the following series?

(a) $\sum_{n=0}^{\infty} c_n 8^n$.

(b) $\sum_{n=0}^{\infty} c_n (-3)^n$.

(c) $\sum_{n=0}^{\infty} (-1)^n c_n 9^n$.
25. If \( \sum_{n=0}^{\infty} c_n 4^n \) is convergent, does it follow that the following series are convergent?

(a) \( \sum_{n=0}^{\infty} c_n (-2)^n \).

(b) \( \sum_{n=0}^{\infty} c_n (3)^n \).

(c) \( \sum_{n=0}^{\infty} c_n (-4)^n \).
26. Find the radius and interval of convergence of the series.

(a) \( \sum_{n=1}^{\infty} \frac{x^n}{n^2} \).

(b) \( \sum_{n=1}^{\infty} \frac{(-1)^n x^n}{n 2^n} \).

(c) \( \sum_{n=0}^{\infty} \frac{n}{4^n} (2x - 1)^n \).
(d) \( \sum_{n=1}^{\infty} (-1)^n \frac{(x-1)^n}{\sqrt{n}} \).

(e) \( \sum_{n=1}^{\infty} \frac{(x-4)^n}{n5^n} \).

(f) \( \sum_{n=0}^{\infty} \frac{2^n(x-3)^n}{n+3} \).
(g) \[ \sum_{n=1}^{\infty} \frac{(x + 1)^n}{n(n+1)}. \]

(h) \[ \sum_{n=1}^{\infty} n!(2x - 1)^n. \]

(i) \[ \sum_{n=1}^{\infty} \frac{n x^n}{1 \cdot 3 \cdot 5 \cdots (2n - 1)}. \]
27. If $k$ is a positive integer, find the radius of convergence of the series
\[
\sum_{n=0}^{\infty} \frac{(n!)^k}{(nk)!} x^n
\].

28. A function $f$ is defined by:
\[
f(x) = 1 + 2x + x^2 + 2x^3 + x^4 + 2x^5 + x^6 + \cdots
\]
That is, its coefficients are $c_{2n} = 1$ and $c_{2n+1} = 2$ for all $n \geq 0$. Find the interval of convergence of the series and find an explicit formula for $f(x)$.

29. If $f(x) = \sum_{n=0}^{\infty} c_n x^n$ where $c_{n+4} = c_n$ for all $n \geq 0$, find the interval of convergence of the series and a formula for $f(x)$. 
30. Find a power series representation for the following functions, and determine the interval of convergence.

(a) $f(x) = \frac{1}{1+4x^2}$.

(b) $f(x) = \frac{1}{x^4 + 16}$.

(c) $f(x) = \frac{1 + x^2}{1 - x^2}$.