1. If \( f(x) = \sum_{n=0}^{\infty} b_n (x - 5)^n \), for all \( x \). Write a formula for \( b_8 \).

2. Find the Maclaurin series for \( f(x) \). Also find the associated radius of convergence.
   (a) \( f(x) = \frac{1}{(1 + x)^2} \).
   (b) \( f(x) = \frac{x}{1 - x} \).

3. Find the Taylor series for \( f(x) \) at the given value \( a \).
   (a) \( f(x) = \ln(x) \quad a = 2 \).
   (b) \( f(x) = \sqrt{x} \quad a = 4 \).
   (c) \( f(x) = \cos(x) \quad a = -\pi/4 \).

4. Use a Maclaurin series developed previously to obtain a Maclaurin Series for the given function.
   (a) \( f(x) = \cos(x^3) \).
   (b) \( f(x) = xe^{-x} \).
   (c) \( f(x) = \sin^2(x) \).

5. Find the Maclaurin series for \( f \) and its radius of convergence.
   (a) \( f(x) = \frac{1}{\sqrt{1 + 2x}} \).
   (b) \( f(x) = (1 + x)^{-3} \).
   (c) \( f(x) = 2^x \).

6. Use the series to approximate the definite integral to within the indicated approximation
   (a) \( \int_{0}^{0.5} \cos(x^2) \, dx \) (Three decimal places).
   (b) \( \int_{0}^{0.1} \frac{1}{\sqrt{1 + x^3}} \, dx \) (error < 10^{-8}).
   (c) \( \int_{0}^{0.5} x^2 e^{-x^2} \, dx \) (error < 0.001).

7. Find the limit \( \lim_{x \to 0} \frac{1 - \cos x}{1 + x - e^x} \).

8. Find the third nonzero terms of the Maclaurin series for each function.
   (a) \( f(x) = e^{-x^2} \cos(x) \).
   (b) \( f(x) = \sec(x) \).

9. Find the sum of the series.
   (a) \( \sum_{n=0}^{\infty} (-1)^n \frac{\pi^{2n+1}}{4^{2n+1}(2n+1)!} \).
   (b) \( f(x) = \sum_{n=2}^{\infty} \frac{x^{3n+1}}{n!} \).
\[ f(x) = \sum_{n=0}^{\infty} \frac{x^{n+1}n}{(n+1)!}. \]

10. (a) Approximate \( f \) by a Taylor polynomial with degree \( n \) at the number \( a \). (b) Use Taylor’s Inequality to estimate the accuracy of the approximation \( f(x) \approx T_n(x) \) when \( x \) lies in the given interval.

(a) \( f(x) = \sin(x) \quad a = \pi/4, \quad n = 5, \quad 0 \leq x \leq \pi/2 \).

(b) \( f(x) = e^{x^2} \quad a = 0, \quad n = 3, \quad 0 \leq x \leq 0.1 \).

(c) \( f(x) = \ln(x) \quad a = 4, \quad n = 3, \quad 3 \leq x \leq 5 \).

11. Estimate \( \sin(35^\circ) \) correct to five decimal places.

12. Use Taylor’s Inequality to determine the number of terms of the Maclaurin Series for \( e^x \) that should be used to estimate \( e^{0.1} \) to within 0.0001.

13. Use the Alternating Series Estimation or Taylor’s Inequality to estimate the range of values of \( x \) for which the given approximation is accurate to within the stated error.

\[ \sin x \approx x - \frac{x^3}{6}, \quad \text{error} < 0.01 \]

14. Find a power series representation for the following functions, and determine the interval of convergence.

(a) \( f(x) = \frac{1}{(1 + x)^3} \).

(b) \( f(x) = x \ln(1 + x) \).

(c) \( f(x) = \frac{x^2}{(1 - 2x)^2} \).

15. Evaluate the indefinite integral as a power series

(a) \( \int \frac{x}{1 + x^3} \, dx \).

(b) \( \int \frac{\tan^{-1}(x)}{x} \, dx \).

(c) \( \int \tan^{-1}(x^2) \, dx \).

16. Use a power series to approximate the definite integral to six decimal places.

(a) \( \int_0^{1/2} \tan^{-1}(x^2) \, dx \).

(b) \( \int_0^{0.5} \frac{1}{1 + x^6} \, dx \).

(c) \( \int_0^{1/3} x^2 \tan^{-1}(x^4) \, dx \).

17. Defined the function \( f \) by the power series

\[ f(x) = \sum_{n=0}^{\infty} \frac{x^n}{n!}. \]

(a) Show that \( f \) is solution of the differential equation

\[ f'(x) - f(x) = 0 \quad \text{for all} \quad x \]

(b) Prove that \( f(x) = e^x \).