1. Determine whether the series is absolutely convergent or not.

(a) \( \sum_{n=1}^{\infty} \frac{\sin(2n)}{n^2} \).

(b) \( \sum_{n=1}^{\infty} \frac{(n+1)5^n}{n^2} \).

(c) \( \sum_{n=1}^{\infty} \frac{(n+2)!}{n!10^n} \).

(d) \( \sum_{n=1}^{\infty} (-1)^n \frac{n}{n^2+1} \).

(e) \( \sum_{n=1}^{\infty} (-1)^n \frac{\ln(n)}{n} \).

(f) \( \sum_{n=1}^{\infty} \frac{(-1)^n}{2^n n!} \).

2. Find the radius of convergence and interval of convergence of the series

(a) \( \sum_{n=0}^{\infty} \frac{x^n}{n+2} \).

(b) \( \sum_{n=0}^{\infty} \frac{n^2 x^n}{10^n} \).

(c) \( \sum_{n=1}^{\infty} \frac{(-1)^n x^{2n-1}}{(2n-1)} \).

(d) \( \sum_{n=0}^{\infty} \frac{(-3)^n x^{2n}}{n+1} \).

(e) \( \sum_{n=1}^{\infty} \frac{x^n}{3^n n^3} \).

(f) \( \sum_{n=1}^{\infty} \frac{2^n (x-3)^n}{\sqrt{n+3}} \).

3. Find a power series representation for the function \( f \).

(a) \( f(x) = \frac{1}{x^2 + 25} \).

(b) \( f(x) = \ln\left(\frac{1+x}{1-x}\right) \).

(c) \( f(x) = \tan^{-1}(2x) \).

4. Starting with the geometric series \( \sum_{n=0}^{\infty} x^n \), find the sum of the following series.

(a) \( \sum_{n=1}^{\infty} nx^{n-1} \), \( \sum_{n=1}^{\infty} nx^n \) \(|x| < 1\).
\[ \sum_{n=1}^{\infty} \sum_{n=2}^{\infty} n(n-1)x^n \] 
\[ |x| < 1. \]
\[ \sum_{n=2}^{\infty} \frac{n^2 - n}{2^n}, \quad \sum_{n=1}^{\infty} \frac{n^2}{2^n}. \]

5. Find the Taylor series for \( f \) at the given value of \( a \).

(a) \( f(x) = e^x \quad a = 3 \).
(b) \( f(x) = \frac{1}{x} \quad a = 1 \).
(c) \( f(x) = \sin(x) \quad a = \pi/4 \).

6. Find the Maclaurin series for \( f \) and its radius of convergence. You may use the direct method or known series.

(a) \( f(x) = \sqrt{1 + x} \).
(b) \( f(x) = 10^x \).
(c) \( f(x) = (1 - 3x)^{-5} \quad a = \pi/4 \).

7. Evaluate the integral \( \int e^x x \) dx as an infinite series.

8. (a) Approximate \( f(x) = \sqrt{x} \) by a Taylor polynomial with degree \( n = 3 \) at the point \( a = 1 \). (b) Use Taylor’s inequality to estimate the accuracy of the approximation \( f(x) \approx T_n(x) \) where \( x \) lies in the interval \( 0.9 \leq x \leq 1.1 \).

9. (a) Approximate \( f(x) = \sec(x) \) by a Taylor polynomial with degree \( n = 2 \) at the point \( a = 0 \). (b) Use Taylor’s inequality to estimate the accuracy of the approximation \( f(x) \approx T_n(x) \) where \( x \) lies in the interval \( 0 \leq x \leq \pi/6 \).

10. Show that the given equation represents a sphere, and find its center and radius.

(a) \( x^2 + y^2 + z^2 + 2x + 8y - 4z = 28 \).
(b) \( x^2 + y^2 + z^2 = 6x + 4y + 10z \).
(c) \( 2x^2 + 2y^2 + 2z^2 + 4y - 2z = 1 \).

11. (a) Find an equation of a sphere if one of its diameters has endpoints \((2, 1, 4) \) and \((4, 3, 10) \).
(b) Find an equation of the sphere that has center \((1, 2, 3) \) and passes through the point \((-1, 1, -2) \).
(c) Find an equation of the sphere with center \((2, -3, 6) \) that touch the \( xy \)-plane.

12. Describe the region of the three-dimensional space described by the equation or inequality

(a) \( x = 9 \).
(b) \( z \leq 0 \).
(c) \( y^2 + z^2 \leq 4 \).
(d) \( 1 \leq x^2 + y^2 + z^2 \leq 25 \).
(e) \( xy = 0 \).
(f) \( xyz = 0 \).