1. If \( f(x) = \sum_{n=0}^{\infty} b_n(x-5)^n \), for all \( x \). Write a formula for \( b_8 \).

2. Find the Maclaurin series for \( f(x) \). Also find the associated radius of convergence.

(a) \( f(x) = \frac{1}{(1+x)^2} \).

(b) \( f(x) = \frac{x}{1-x} \).
3. Find the Taylor series for $f(x)$ at the given value $a$.

(a) $f(x) = \ln(x) \quad a = 2$.

(b) $f(x) = \sqrt{x} \quad a = 4$.

(c) $f(x) = \cos(x) \quad a = -\pi/4$. 
4. Use a Maclaurin series developed previously to obtain a Maclaurin Series for the given function.
   
   (a) \( f(x) = \cos(x^3) \).

   (b) \( f(x) = xe^{-x} \).

   (c) \( f(x) = \sin^2(x) \).
5. Find the Maclaurin series for \( f \) and its radius of convergence.

(a) \( f(x) = \frac{1}{\sqrt{1 + 2x}} \).

(b) \( f(x) = (1 + x)^{-3} \).

(c) \( f(x) = 2^x \).
6. Use the series to approximate the definite integral to within the indicated approximation

(a) $\int_0^{0.5} \cos(x^2)\,dx$ (Three decimal places).

(b) $\int_0^{0.1} \frac{1}{\sqrt{1 + x^3}}\,dx$ (error < $10^{-8}$).

(c) $\int_0^{0.5} x^2e^{-x^2}\,dx$ (error < 0.001).
7. Find the limit \( \lim_{{x \to 0}} \frac{1 - \cos x}{1 + x - e^x} \).

8. Find the third nonzero terms of the Maclaurin series for each function.

(a) \( f(x) = e^{-x^2} \cos(x) \).

(b) \( f(x) = \sec(x) \).
9. Find the sum of the series.

(a) \[ \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{4^{2n+1}(2n+1)!} \cdot \]

(b) \[ f(x) = \sum_{n=2}^{\infty} \frac{x^{3n+1}}{n!} \cdot \]

(c) \[ f(x) = \sum_{n=0}^{\infty} \frac{x^{n+1}n}{(n+1)!} \cdot \]
10. (a) Approximate $f$ by a Taylor polynomial with degree $n$ at the number $a$. (b) Use Taylor’s Inequality to estimate the accuracy of the approximation $f(x) \approx T_n(x)$ when $x$ lies in the given interval.

(a) $f(x) = \sin(x) \quad a = \pi/4, \quad n = 5, \quad 0 \leq x \leq \pi/2$.

(b) $f(x) = e^{x^2} \quad a = 0, \quad n = 3, \quad 0 \leq x \leq 0.1$.

(c) $f(x) = \ln(x) \quad a = 4, \quad n = 3, \quad 3 \leq x \leq 5$. 
11. Estimate $\sin(35^\circ)$ correct to five decimal places.

12. Use Taylor’s Inequality to determine the number of terms of the Maclaurin Series for $e^x$ that should be used to estimate $e^{0.1}$ to within 0.0001.

13. Use the Alternating Series Estimation or Taylor’s Inequality to estimate the range of values of $x$ for which the given approximation is accurate to within the stated error.

$$\sin x \approx x - \frac{x^3}{6}, \quad \text{error} < 0.01$$