1. (10 pts) (Van der Pol oscillator) Determine nullclines and draw phase portrait of the van der Pol oscillator (given in the Lienard (1928) form)
\[
\begin{align*}
\dot{x} &= x - x^3/3 - y \\
\dot{y} &= bx
\end{align*}
\]

2. (10 pts) (Bonhoeffer-van der Pol oscillator) Determine nullclines and sketch representative phase portraits of the Bonhoeffer-van der Pol oscillator.
\[
\begin{align*}
\dot{x} &= x - x^3/3 - y \\
\dot{y} &= b(x - a) - cy
\end{align*}
\]
in the case of \(c = 0\). Treat \(a\) and \(b > 0\) as parameters.

3. (10 pts) (Hindmarsh-Rose spiking neuron) The following system is a generalization of the FitzHugh-Nagumo model (Hindmarsh and Rose 1982)
\[
\begin{align*}
\dot{x} &= f(x) - y + I \\
\dot{y} &= g(x) - y
\end{align*}
\]
where \(f(x) = -ax^3 + bx^2\), \(g(x) = -c + dx^2\), and \(a, b, c, d,\) and \(I\) are parameters. Suppose \((\bar{x}, \bar{y})\) is an equilibrium. Determine its type and stability as a function of \(f' = f'(\bar{x})\) and \(g' = g'(\bar{x})\); that is, plot a diagram similar to the one in the figure below with \(f'\) and \(g'\) as coordinates.
4. (10 pts) (I\(_K\)-model) Show that the unique equilibrium in the I\(_K\)-model

\[
\begin{align*}
\dot{C} & = -g_L(V - E_L) - g_K m^4(V - E_K) \\
\dot{m} & = (m_\infty(V) - m)/\tau(V)
\end{align*}
\]

is always stable, at least when \(E_L > E_K\). (Hint: look at the signs of trace and determinant of the Jacobian matrix).

5. (10 pts) (I\(_h\)-model) Show that the unique equilibrium in the I\(_h\)-model

\[
\begin{align*}
\dot{C} & = -g_L(V - E_L) - g_h h(V - E_K) \\
\dot{h} & = (h_\infty(V) - h)/\tau(V)
\end{align*}
\]

is always stable.

6. (10 pts) (Bendixson’s criteria) If the divergence of vector field

\[
\frac{\partial f}{\partial x} + \frac{\partial g}{\partial y}
\]

of a two-dimensional dynamical system is not identically zero and does not change sign on the plane, then the dynamical system cannot have limit cycles. Use this criterion to show that the I\(_K\)-model and the I\(_h\)-model cannot oscillate.

7. (10 pts) Determine stability of equilibria in the following model

\[
\begin{align*}
\dot{x} & = a + x^2 - y \\
\dot{y} & = bx - cy
\end{align*}
\]

where \(a \in \mathbb{R}, b \geq 0\) and \(c > 0\) are some parameters.