Disclaimer: This review is just a selection of good problems on the main topics in our course. It is absolutely NOT meant as a preview of the final exam or as a sample exam. Any problem from, or similar to, quizzes, midterms, homework, classroom, WIR and suggested problems could appear on the final exam.

1. A quality colored pencils manufacturer determines that when the price of a box of pencils is $15.00, the quantity demanded is 1200. When the price is $22.50, the quantity demanded decreases by 100. The supplier is not willing to supply any pencil boxes at a price of $10.00, but will supply 300 pencil boxes at a price of $17.50. What is the market equilibrium for this quality pencil manufacturer?

2. Show all the steps of the Gauss-Jordan elimination method that transform the matrix below to row-reduced form.

\[
\begin{pmatrix}
3 & -6 & 15 \\
4 & 2 & -6
\end{pmatrix}
\]
3. Solve the following systems of linear equations:

\[
\begin{align*}
-x + 3y + 2z &= 1 \\
x - 2y + z &= -5 \\
2x - y &= 11
\end{align*}
\]

\[
\begin{align*}
2x + 4y - 3z &= 10 \\
-x + 2y - 3z &= 4 \\
3x + 6y + 9z &= 12
\end{align*}
\]

\[
\begin{align*}
x - 2y + 3z &= -4 \\
x + 3y - 2z &= 1
\end{align*}
\]

\[
\begin{bmatrix}
-1 & 3 & 2 \\
1 & -2 & 1 \\
2 & -1 & 0
\end{bmatrix}
\begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
2 & 4 & 3 \\
-1 & 2 & -3 \\
3 & -6 & 9
\end{bmatrix}
= \begin{bmatrix}
0 & 4 & 3 \\
0 & 2 & -3 \\
0 & -6 & 12
\end{bmatrix}
\]

Do \text{rref} \begin{bmatrix}
1 & 0 & 0 & 8 \\
0 & 1 & 0 & 5 \\
0 & 0 & 1 & -3
\end{bmatrix} = \begin{bmatrix}
1 & 0 & 0 & 8 \\
0 & 1 & 0 & 5 \\
0 & 0 & 1 & -3
\end{bmatrix}

Unique solution: \(x = 8, y = 5, z = -3\)

4. Solve the following matrix equation for \(a, b, c\):

\[
\begin{bmatrix}
2.5 & 0.5b \\
1.5c & 5
\end{bmatrix}
-\begin{bmatrix}
a & -b \\
-a & 4c
\end{bmatrix}
= \begin{bmatrix}
8 & 1 \\
7 & 6
\end{bmatrix}
\]

\[
\begin{bmatrix}
5 & b \\
-3c & 10
\end{bmatrix}
-\begin{bmatrix}
a & -a \\
-b & 4c
\end{bmatrix}
= \begin{bmatrix}
8 & 1 \\
7 & 6
\end{bmatrix}
\]

\[
5 - a = 8 \Rightarrow a = 5 - 8 = -3
\]

\[
b + a = 1 \Rightarrow b = 1 - a = 1 - (-3) = 4
\]

\[
3c + b = 7 \Rightarrow 3c = 7 - b = 7 - 4 = 3 \Rightarrow c = 1
\]

\[
10 - 4c = 6 \quad 10 - 4 \cdot 1 = 6 \checkmark
\]
5. A toy factory makes two kinds of wooden puzzles: dolls and trains. Each doll requires 6 pieces of wood and takes 8 hours to make. Each train requires 4 pieces of wood and takes 2 hours to make. The profit on each doll is $12, and on each train $3. It is determined that the number of trains made should be at most twice the number of dolls made. The toy factory only has 42 pieces of wood available each week, but can afford at least at least 24 hours of work each week. How many dolls and trains should the factory make each week in order to maximize profit? Is there any leftover wood at this level of production? What would be the level of production that minimizes profit?

\[ \begin{array}{|c|c|c|c|} 
\hline 
\text{pieces} & \text{hrs} & \text{Profit} & \text{Function} \\
\hline 
\text{x=dolls} & 6 & 8 & $12 \\
\text{y=trains} & 4 & 2 & $3 \\
\hline 
6x+4y & 8x+2y & 12x+3y \\
\hline 
\end{array} \]

Let \( x = \# \text{ of dolls}, \ y = \# \text{ of trains} \)

\[ \begin{align*}
6x+4y & \leq 42 \\
8x+2y & \geq 24 \\
y & \leq 2x \\
x & \geq 0, \ y \geq 0
\end{align*} \]

\( B \) \quad \begin{align*}
3x+2y & \leq 21 \\
4x+y & \geq 12
\end{align*} \]

\( R \) \quad \begin{align*}
4x+y & \geq 12 \\
y & \leq 2x
\end{align*} \]

\( B \) \quad \begin{align*}
3x+2y & = 21 \\
y & = 2x
\end{align*} \]

\( x \geq 0, \ y \geq 0 \)

Within our feasible region, the min. profit occurs at all points of the segment from \((2, 4)\) to \((3, 0)\).

\[ \begin{align*}
\text{pts} & + \text{Profit} = 12x+3y \\
\text{1st} & \quad 2, 4 & 12+4=16 \quad 24+12=36 \\
\text{2nd} & \quad 3, 6 & 36+18=54 \\
\text{3rd} & \quad 3, 0 & 36 \\
\text{4th} & \quad 7, 0 & 84 \quad \text{maximum profit: 7 dolls, 0 trains.}
\end{align*} \]

Wood used: \( 7 \times 6 = 42 \), no wood left.
6. A survey of 60 customers of an ice cream parlor was done asking which toppings were their favorite, among fruit (F), nuts (N) and sprinkles (S). Six customers only liked nuts, 32 liked sprinkles, 7 liked all three, 7 liked fruit and sprinkles but not nuts, 13 liked nuts and sprinkles, 30 customers liked exactly one of the toppings, 21 liked fruit or nuts but not sprinkles.

a. How many customers liked none of the toppings?

\[ 60 - 21 - 20 - 12 = 7 \]

b. How many customers liked fruit?

\[ 12 + 3 + 7 + 7 = 29 \]

c. What is \( n((N \cap F) \cup (S \cap N')) \)?

\[ 7 + 12 + 3 + 7 = 29 \]

d. What is the probability that a customer in this group liked exactly 2 of the toppings?

\[ \frac{n(\text{those who liked 2 toppings})}{n(S)} = \frac{3 + 7 + 6}{60} = \frac{16}{60} = \frac{4}{15} \]

7. A bag contains 10 number magnets and 11 letter magnets. Of the numbers, 2 are red, 7 yellow and 1 green. Of the letters, 3 are red, 4 are yellow and 4 are green. An experiment consists of selecting a magnet from the bag.

a. What is the probability that a green magnet or a letter is selected.

\[ P(G \cup \text{letter}) = P(G) + P(\text{letter}) - P(\text{green letter}) = \frac{5 + 11 - 4}{21} = \frac{12}{21} = \frac{4}{7} \]

b. What is the probability that a red magnet is not selected.

\[ P(\text{not red}) = \frac{21 - 5}{21} = \frac{16}{21} \]

c. Let \( E \) be the event that a yellow magnet is selected. Let \( F \) be the event that a number is selected. Are \( E \) and \( F \) mutually exclusive? Are \( E \) and \( F \) independent?

Not mutually exclusive because there are yellow numbers.

\[ P(\text{yellow number}) = \frac{3}{21} = \frac{1}{7} \]

\[ P(\text{yellow}) = \frac{10}{21} \]

\[ P(E) = P(\text{yellow}) = \frac{11}{21} \]

\[ P(F) = \frac{0}{21} \]

NOT INDEP. b/c \( \frac{10}{21} \cdot \frac{1}{7} \neq \frac{1}{3} \).
8. A bag contains domino tiles: 10 are blue, 9 green, 8 red, and 7 white. A sample of 8 tiles is taken from the bag without looking. What is the probability that the sample contains

a. exactly 3 green domino tiles or exactly 4 white ones?

\[
P(3 \text{ green}) + P(4 \text{ white}) - P(3 \text{ green and 4 white})
\]

\[
= \frac{C(9,3)C(25,5)+C(7,4)C(27,4)-C(9,3)C(7,4)C(18,1)}{C(34,8)}
\]

b. at least 1 red domino tile?

\[
1 - P(\text{no red}) = 1 - \frac{C(26,8)}{C(34,8)}
\]

9. A group of 55 4th grade students is made up of 32 girls, the rest boys. Of the girls, 18 play sports on weekends. Of the boys 20 play sports on weekends. If a randomly selected 4th grade student from this group does not play sports on weekends, what is the probability that the student selected is a boy?

\[
P(\text{boy|no sports on weekends}) = \frac{P(B \cap \neg S)}{P(S)}
\]

\[
= \frac{\frac{3}{23} \cdot \frac{23}{55}}{\frac{14}{32} \cdot \frac{32}{55} + \frac{11}{32} \cdot \frac{32}{55}}
\]

10. Paul pays $3 to play a game of dice with Emma. The game consists of Paul rolling the dice: if the sum of the uppermost numbers is a 5, Paul wins $10 from Emma. If exactly one five is rolled, Paul wins $7 from Emma, and if a double is rolled, Paul wins $4 from Emma. Otherwise, Paul pays $1 to Emma. Is this a fair game? If not, what should Paul pay for the game to be fair?

\[
\text{Expected net win} = 7 \cdot \frac{4}{36} + 10 \cdot \frac{1}{36} + 16 \cdot \frac{1}{36} - \frac{7}{9} + \frac{10}{9} + \frac{1}{6} - \frac{16}{9} = \frac{7}{9} + \frac{10}{9} + \frac{1}{6} - \frac{16}{9} = \frac{1}{9} + \frac{1}{6} - 0 \not= 0
\]

expected win = 0.2767
In order for game to be fair, set 
\[
(10-A) \frac{4}{36} + (7-A) \left( \frac{10}{36} \right) + (4-A) \left( \frac{6}{36} \right) + (-A-1) \frac{16}{36} = 0
\]

\[
\frac{40 - 4A}{36} + \frac{70}{36} - \frac{10A}{36} + \frac{24}{36} - \frac{6A}{36} - \frac{16A}{36} - \frac{16}{36} = 0
\]

\[
\frac{40 + 70 + 24 - 16}{36} = \frac{4 + 10 + 6 + 6}{36} \]

118 = 36 A

A = \frac{118}{36} = 3.27

11. If \( P(E) = 0.44 \), \( P(E^c \cup F) = 0.77 \), \( P(F) = 0.50 \), what is \( P(E^c \cap F) \)?

\[
P(E^c \cup F) = P(E^c) + P(F) - P(E^c \cap F)
\]

0.77 = 0.56 + 0.50 - P(E^c \cap F)

P(E^c \cap F) = 0.56 + 0.50 - 0.77 = 0.29

0.50 - 0.29 = 0.21

0.44 - 0.21 = 0.23

12. A flashlight manufacturing company has estimated that 19% of their flashlights will last less than one year. In a shipment of 300 flashlights, what is the probability that more than 20 will last less than a year?

\[
\text{Success: flashlight lasts < 1yr.}
\]

\[
P(\text{success}) = 0.19
\]

\[
1 - \text{binomcdf}(300,0.19,20)
\]

13. A gadget company incurs monthly costs of $33000. Each gadget costs the company $9 to make and is sold for $15. What is the break-even point for this company? Round to the nearest number of gadgets and to the nearest cent.

\[
\text{Rev} = \text{Cost}
\]

\[
15X = 33000 + 9X
\]

\[
6X = 33000
\]

\[
X = \frac{33000}{6} = 5500 \text{ gadgets}
\]
14. Let $U = \{1, 2, 3, 4, 5, 6\}$, $A = \{1, 3, 5\}$, $B = \{2, 3, 4\}$, and $C = \{2, 4, 5\}$. Find the following sets.

a. $(A \cup B^c)^c = A^c \cap B^c = A^c \cap B - \{2, 4\}$

$b. (B \cup C)^c = B^c \cap C^c$

$c. B \cap (A \cup C)^c = B \cap (A^c \cap C^c) = B \cap \{6\} = \emptyset$

15. An office machine is purchased new for $30,800 and it will have a value of $15,400 after five years. Find the equation for depreciation, and determine how many years after purchase the machine will be worth $6,160.

\[
(0, 30,800) \quad (5, 15,400)
\]

\[
m = \frac{\Delta y}{\Delta x} = \frac{15,400 - 30,800}{5 - 0} = -\frac{15,400}{5} = -3,080
\]

\[
y - y_o = m(x - x_o)
\]

\[
y - 30,800 = -3,080(x - 0)
\]

\[
V = 30,800 - 3,080t
\]

\[
6,160 = 30,800 - 3,080t
\]

\[
6,160 - 30,800 = -3,080t
\]

\[
t = \frac{6,160 - 30,800}{-3,080} = 8 \text{ yr.}
\]
A farmer has 150 total acres on which to plant corn and cotton. Because of demand, the farmer knows to plant more corn than cotton (or at least the same amounts). Also, the farmer has budgeted $540 for seeds that cost $3/acre for corn and $5/acre for cotton. For this situation, define the variables and then write a system of inequalities to describe the constraints. Next, graph the feasible region for the possible numbers of acres on which to plant corn and cotton. At harvest time, each acre of corn yields a profit of $80, and each acre of cotton yields a profit of $120. How many acres of each should the farmer plant in order to maximize profit? **Answer:** 105 acres of corn and 45 acres of cotton for a maximum of $13,800.
16. Which, if any, of the matrices below is in rref? If a matrix is not, complete the necessary steps to get to rref.

\[
\begin{bmatrix}
1 & -3 & 4 & 1 \\
0 & 1 & -2 & 2 \\
0 & 0 & 0 & 3 \\
\end{bmatrix}
\quad \begin{bmatrix}
1 & 4 & 5 & -9 \\
0 & 2 & 4 & -6 \\
0 & 0 & 0 & -5 \\
\end{bmatrix}
\quad \begin{bmatrix}
1 & 1 & 3 & 3 \\
0 & 1 & 0 & 4 \\
0 & 0 & 0 & 1 \\
\end{bmatrix}
\]

R₁ → R₁ + 3R₂
\[
\begin{bmatrix}
1 & -3 & 4 & 1 \\
0 & 1 & -2 & 2 \\
0 & 0 & 0 & 3 \\
\end{bmatrix}
\]

\[
\begin{array}{c}
\frac{1}{2} R₂ \\
\frac{1}{3} R₃ \\
\end{array}
\]

- \frac{7}{5} R₃

17. Andrea, a self-employed individual, wishes to accumulate a retirement fund of $250,000. How much should she deposit each month into her retirement account, which pays interest at a rate of 5.5%/year compounded monthly, to reach her goal upon retirement 25 years from now? (Round your answer to the nearest cent.)
18. A corporation creates a sinking fund in order to have $380,000 to replace some machinery in 8 years. How much should be placed in this account at the end of each week if the annual interest rate is 8.7% compounded weekly? (Round your answers to the nearest cent.)

a. How much interest would they earn over the life of the account?

b. Determine the value of the fund after 2, 4, and 6 years.

2 years

4 years

6 years

19. Carl is the beneficiary of a $25,000 trust fund set up for him by his grandparents. Under the terms of the trust, he is to receive equal installments from this fund at the end of each year over a 7-year period. If the fund earns interest at the rate of 9%/year compounded annually, what amount will he receive each year? (Round your answer to the nearest cent.)
20. Find the monthly payment needed to amortize a typical $100,000 mortgage loan amortized over 30 years at an annual interest rate of 4.7% compounded monthly. (Round your answers to the nearest cent.) Find the total interest paid on the loan.

21. A group of private investors purchased a condominium complex for $4 million. They made an initial down payment of 12% and obtained financing for the balance. The loan is to be amortized over 15 years at an interest rate of 15% per year compounded quarterly. (Round your answers to the nearest cent.)

   a. What is the required quarterly payment?

   b. How much total interest will be paid on the loan?
22. Five years ago, Diane secured a bank loan of $300,000 to help finance the purchase of a loft in the San Francisco Bay area. The term of the mortgage was 30 years, and the interest rate was 9% per year compounded monthly on the unpaid balance. Because the interest rate for a conventional 30-year home mortgage has now dropped to 5% per year compounded monthly, Diane is thinking of refinancing her property. (Round your answers to the nearest cent.)

   a. What is Diane's current monthly mortgage payment?

   b. What is Diane's current outstanding balance?

   c. If Diane decides to refinance her property by securing a 30-year home mortgage loan in the amount of the current outstanding principal at the prevailing interest rate of 5% per year compounded monthly, what will be her monthly mortgage payment? Use the rounded outstanding balance.

   d. How much less would Diane's monthly mortgage payment be if she refinances? Use the rounded values from parts (a)-(c).
23. The Martinezes are planning to refinance their home (assuming that there are no additional finance charges). The outstanding balance on their original loan is $200,000. Their finance company has offered them two options:

| Option A: A fixed-rate mortgage at an interest rate of 6.5% per year compounded monthly, payable over a 25-year period in 300 equal monthly installments. | Option B: A fixed-rate mortgage at an interest rate of 6.25% per year compounded monthly, payable over a 12-year period in 144 equal monthly installments. |

a. Find the monthly payment required to amortize each of these loans over the life of the loan. (Round your answers to the nearest cent.)

   Option A: 

   Option B: 

b. How much interest would the Martinezes save if they chose the 12-year mortgage instead of the 25-year mortgage?
   Use the rounded monthly payment values from part (a). (Round your answer to the nearest cent.)

24. The Turners have purchased a house for $170,000. They made an initial down payment of $10,000 and secured a mortgage with interest charged at the rate of 10%/year compounded monthly on the unpaid balance. The loan is to be amortized over 30 yr. (Round your answers to the nearest cent.)

   a. What monthly payment will the Turners be required to make?

   b. How much total interest will they pay on the loan?

   c. What will be their equity after 10 years?

   d. What will be their equity after 22 years?