This last WIR is based on homework problems.

Section 8.5 - Round answers to four decimal places.

1. a. Choose a sketch of the area under the standard normal curve corresponding to \( P(0.6 < Z < 1.8) \).

\[ \text{a.} \quad \quad \text{b.} \quad \quad \text{c.} \quad \quad \text{d.} \]

\[ \text{b.} \quad \text{Find the value of } P(0.6 < Z < 1.8). \quad \sim 0.2383 \]

\[ \text{normalcdf}(0.6, 1.8, 0, 1) \]

\[ \text{c.} \quad \text{Find the probability } P(Z \geq -0.65) \text{ given that } Z \text{ is the random variable with standard normal distribution.} \]

\[ 1 - P(Z < -0.65) = 1 - \text{normalcdf}(-99, -0.65, 0, 1) \sim 0.7422 \]
3. Suppose $X$ is a normal random variable with $\mu = 380$ and $\sigma = 21$. Find the following probabilities.
   
   a. $P(X < 388) = \text{normal cdf}(-99, 388, 380, 21) \approx 0.6484$
   
   b. $P(388 < X < 407) = P(X \leq 407) - P(X < 388) = \text{normal cdf}(388, 407, 380, 21) \approx 0.2523$
   
   c. $P(X > 405) = 1 - P(X \leq 405) = 1 - \text{normal cdf}(-999, 405, 380, 21) \approx 0.1169$

4. Let $Z$ be the standard normal variable. Find the values of $z$ that satisfy the given probabilities.
   
   a. $P(Z > z) = 0.2663$

   b. $P(Z < z) = 0.1079 = \text{inverse Normal}(0.1079, 0, 1) \approx -1.2378$

   c. $P(-z < Z < z) = 0.3962 = 1 - 0.3962 = 0.6038$

   Area on each side $= 0.6038 \div z = \frac{0.6038}{z} = 0.3019$

   $z = \text{invNorm}(0.3019 + 0.3962, 0, 1) \approx 0.5189$
5. Find the indicated quantities given that $X$ is a normal random variable with a mean of 20 and a standard deviation of 4.

a. Find the value of \( b \) such that $P(X \leq b) = 0.9463$

$$\text{invNorm}(0.9463, 20, 4) \approx 26.44$$

b. Find the value of \( c \) such that $P(X \geq c) = 0.6664$

Then $P(X < c) = 0.3336 = 1 - 0.6664$

$$\text{invNorm}(0.3336, 20, 4) \approx 18.28$$

c. Find the values of \( A \) and \( B \) such that $P(A \leq X \leq B) = 0.9643$ if \( A \) and \( B \) are symmetric about the mean.

\[
\begin{align*}
1 - .9643 &= .0357 \\
.0357 \div 2 &= .01785 \\
B &= \text{invNorm}(.01785 + .9643, 20, 4) \approx 28.40
\end{align*}
\]

\[
\begin{align*}
A &= 20 - 8.4 = 11.6, \\
B &= 28.40
\end{align*}
\]
1. On the average, a student takes 93 words/minute midway through an advanced court reporting course at the American Institute of Court Reporting. Assuming that the dictation speeds of the students are normally distributed and that the standard deviation is 19 words/minute, find the probability that a student randomly selected from the course can make dictation at the following speeds. (Give your answers to four decimal places.)

a. more than 131 words/minute

\[ P(X > 131) = 1 - P(X \leq 131) = 1 - \text{normalcdf}(-999, 131, 93, 19) \approx 0.0228 \]

b. between 112 and 131 words/minute

\[ P(112 \leq X \leq 131) = \text{normalcdf}(112, 131, 93, 19) \approx 0.1359 \]

c. less than 55 words/minute

\[ P(X < 55) = \text{normalcdf}(-999, 55, 93, 19) \approx 0.0228 \]

2. The weight of topsoil sold in a week is normally distributed with a mean of 400 tons and a standard deviation of 20 tons.

a. What percentage of weeks will sales exceed 440 tons? (Round your answer to two decimal places.)

\[ P(X > 440) = 1 - P(X \leq 440) = 1 - \text{normalcdf}(-999, 440, 400, 20) \approx 0.0228 \]

b. What percentage of weeks will sales be less than 380 tons? (Round your answer to two decimal places.)

\[ P(X < 380) = \text{normalcdf}(-999, 380, 400, 20) \approx 0.1587 \]

c. What percentage of weeks will sales be between 380 and 410 tons? (Round your answer to two decimal places.)

\[ P(380 \leq X \leq 410) = \text{normalcdf}(380, 410, 400, 20) \approx 0.5328 \]
3. A teacher wishes to "curve" a test whose grades were normally distributed with a mean of 71 and standard deviation of 15. The top 15% of the class will get an A, the next 25% of the class will get a B, the next 30% of the class will get a C, the next 25% of the class will get a D and the bottom 5% of the class will get an F. Find the cutoff for each of these grades. (Round your answers to two decimal places.)

a. The A cutoff is a grade of
   \[ A = \text{invNorm}(0.85, 71, 15) \approx 86.55 \]

b. The B cutoff is a grade of
   \[ B = \text{invNorm}(0.6, 71, 15) \approx 74.8 \]

c. The C cutoff is a grade of
   \[ C = \text{invNorm}(0.3, 71, 15) \approx 63.13 \]

d. The D cutoff is a grade of
   \[ D = \text{invNorm}(0.05, 71, 15) \approx 46.33 \]

4. TKK Products manufactures 50-, 60-, 75-, and 100-watt electric light bulbs. Laboratory tests show that the lives of these light bulbs are normally distributed with a mean of 800 hr and standard deviation of 125 hr. What is the probability that a TKK light bulb selected at random will burn for the following times? (Round your answers to four decimal places.)

a. more than 950 hr
   \[ P(X > 950) = 1 - P(X \leq 950) = 1 - \text{normalcdf}(999, 950, 800, 125) \approx 0.1151 \]

b. less than 650 hr
   \[ P(X < 650) = \text{normalcdf}(-999, 650, 800, 125) \approx 0.1151 \]

c. between 800 and 950 hr
   \[ P(800 < X < 950) = \text{normalcdf}(800, 950, 800, 125) \approx 0.3849 \]

d. between 650 and 850 hr
   \[ \approx 0.5404 \]
5. The distribution of heights of adult males is normally distributed with mean 67 inches and standard deviation 2.7 inches. Answer the following. (Round your answers to 2 decimal places)

a. What minimum height is taller than 68% of all adult males?

\[ \text{invNorm}(0.68, 67, 2.7) \approx 68.26 \text{ in} \]

b. What 2 heights that are symmetric about the mean enclose the middle 78% of all heights?

\[ 67 - 3.31 = 63.69 \text{ in} \]
\[ 67 + 3.31 = 70.31 \text{ in} \]

5.1 –Round your answers to the nearest cent

1. Find the accumulated amount at the end of 10 months on a $1800 bank deposit paying simple interest at a rate of 6%/year.

\[ A = P(1 + rt) = 1800 \left(1 + 0.06 \times \frac{10}{12}\right) = 1890 \]

\[ P = 1800 \]
\[ r = 0.06 \]
\[ t = \frac{10}{12} \text{ yr} \]

2. A bank deposit paying simple interest at the rate of 6%/year grew to a sum of $1300 in 8 months. Find the principal.

\[ 1300 = P \left(1 + 0.06 \times \frac{8}{12}\right) \Rightarrow P = \frac{1300}{1 + 0.06 \times \frac{8}{12} \right} = 1250 \]

3. Determine the simple interest rate at which $1200 will grow to $1274 in 8 months.

\[ I = Prt \]
\[ 74 = 1200 \cdot r \cdot \frac{8}{12} \Rightarrow r = \frac{74 \cdot \frac{12}{8}}{1200} = \frac{74}{1200} \cdot \frac{12}{8} = 0.0925 \]

\[ = 9.25\% \]
4. A young man is the beneficiary of a trust fund established for him 16 yr ago at his birth. If the original amount placed in trust was $20,000, how much will he receive if the money has earned interest at the rate of 10%/year compounded as follows?

   a. annually
   \[ A = 20000 \left(1 + \frac{0.1}{1}\right)^{1 \times 16} = 20000 \left(1 + 0.1\right)^{16} \approx 91899.46 \]

   b. quarterly
   \[ A = 20000 \left(1 + \frac{0.1}{4}\right)^{4 \times 16} \approx 97130.89 \]

   c. monthly
   \[ A = 20000 \left(1 + \frac{0.1}{12}\right)^{12 \times 16} \approx 98406.06 \]

5. Find the present value of $40,000 due in 4 years at the rate of interest of 10%/year compounded daily.

   \[ A = P \left(1 + \frac{r}{m}\right)^{mt} \]
   \[ 40000 = P \left(1 + \frac{0.1}{365}\right)^{365 \times 4} \]
   \[ P = \frac{40000}{\left(1 + \frac{0.1}{365}\right)^{365 \times 4}} \approx 26814.27 \]

6. Five and a half years ago, Chris invested $10,000 in a retirement fund that grew at the rate of 11.14%/year compounded quarterly. What is his account worth today?

   \[ A = 10000 \left(1 + \frac{0.1114}{4}\right)^{4 \times 5.5} \approx 18300.13 \]
7. Kim invested a sum of money 3 yr ago in a savings account that has since paid interest at the rate of 6.5%/year compounded monthly. Her investment is now worth $24,293.43. How much did she originally invest? (Solve in two ways)

\[ 24293.43 = P \left(1 + \frac{0.065}{12}\right)^{12 \times 3} \]

Do by TVM, check answer.

8. Your rich uncle has just given you a high school graduation present of $1,600,000. The present, however, is in the form of a 30-year bond with an annual interest rate of 7.9% compounded annually. The bond says that it will be worth $1,600,000 in 30 years. What is this gift worth at the present time?

\[ 1600000 = P \left(1 + \frac{0.079}{1}\right)^{1 \times 30} \]

\[ P = \frac{1600000}{1.079^{30}} \approx \$16,348,452 \]
9. Find the effective rate of interest corresponding to a nominal rate of 12%/year compounded in the following ways.

a. compounded annually

\[ \text{Eff} = \left(1 + \frac{.12}{1}\right)^1 - 1 = 1.12 - 1 = \frac{12}{100} \]

\[ 12 \text{%} \]

b. compounded semiannually

\[ \text{Eff} = \left(1 + \frac{.12}{2}\right)^2 - 1 = \left(1 + .06\right)^2 - 1 \approx 1.1236 \]

\[ 12.36 \text{%} \]

c. compounded quarterly

\[ \text{Eff} = \left(1 + \frac{.12}{4}\right)^4 - 1 = \left(1 + .03\right)^4 - 1 \approx 1.1255 \]

\[ 12.55 \text{%} \]

d. compounded monthly

\[ \text{Eff} = \left(1 + \frac{.12}{12}\right)^{12} - 1 = \left(1 + .01\right)^{12} - 1 \approx 1.1268 \]

\[ 12.68 \text{%} \]

10. Find the accumulated amount after 3 yr if $5600 is invested at 6%/year compounded continuously.

\[ A = P e^{rt} \]

\[ A = 5600 e^{.06 \times 3} \]

\[ \approx 6704.42 \]