1. A company will not produce any gym shoes for less than $120 a pair. For $145 a pair, they are willing to make 600 pairs of shoes. Find the linear supply function. If the price-demand function is given by \( y = -0.5x + 250 \), find and discuss the equilibrium price and quantity for the shoes.

\[
\text{Slope} = \frac{\Delta y}{\Delta x} = \frac{145 - 120}{600 - 0} = \frac{25}{600} = \frac{1}{24}
\]

\[
y - y_0 = m(x - x_0)
\]

\[
y - 120 = \frac{1}{24}(x - 0)
\]

\[
\begin{align*}
\text{Suppy} & : y = 120 + \frac{1}{24}x \\
\text{Demand} & : y = -0.5x + 250
\end{align*}
\]

\[
120 + \frac{1}{24}x = -\frac{1}{2}x + 250
\]

\[
\frac{1}{24}x + \frac{1}{2}x = 250 - 120
\]

\[
\frac{1 + 12}{24}x = 130
\]

\[
\frac{13}{24}x = 130
\]

\[
x = 130 \cdot \frac{24}{13} = 240 \text{ pairs of shoes.}
\]
2. Jeff’s inbox currently has 98 unread e-mails, and this number is decreasing by 5 per day. Model the number of e-mails as a function of time, measured in days.

\[ n(0) = 98 \]
\[ n(1) = 93 \]
\[ n(2) = 88 \]

\[ n(t) = 98 - 5t \]

3. A company that prints paperback mystery novels has fixed costs of $3600 per month, and $3.50 for each novel printed. Give the linear cost as a function of the number \( x \) of novels printed. The company sells the paperbacks to a retailer for $6.50 each. Give the linear revenue function \( R(x) \). Find and discuss the break-even point.

Cost \( C(x) = 3600 + 3.50x \)

Revenue \( R(x) = 6.50x \)

Break-even pt: \( 3600 + 3.50x = 6.50x \)

\[ 3600 = 6.50x - 3.50x = 3.00x \]

\[ x = \frac{3600}{3} = 1200 \text{ novels printed} \]

\[ R(1200) = C(1200) = 7800 \]
4. A gadget manufacturer has cost function given by \( C(x) = 0.02x^2 + 7.5x + 600 \) dollars, and sells the gadgets for $20 each. Find the revenue function and the break-even point.

Round your answer to the nearest whole number if necessary.

\[
R(x) = 20x
\]

B. E. point: Profit = 0, so \( C = R \)

\[
0.02x^2 + 7.5x + 600 = 20x
0.02x^2 + (7.5 - 20)x + 600 = 0
0.02x^2 - 12.5x + 600 = 0
\]

Solve for \( x \)

\[
a = 0.02 \\
b = -12.5 \\
c = 600
\]

\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

\[
x = \frac{12.5 \pm \sqrt{12.5^2 - 4(0.02)(600)}}{2 \times 0.02}
\]

\[
x = \frac{12.5 \pm \sqrt{108.25}}{0.04} = \frac{12.5 \pm 10.4043}{0.04}
\]

\[
= \begin{cases} 572.608 & \text{to nearest integer} \\ 52.392 \end{cases}
\]

If given \( 10 \leq x \leq 100 \) then only accept 52 gadgets.
5. Danny has to replace a piece of equipment in his shop. He can buy an Alpha machine that costs $125,820 and takes $83 dollars to maintain each week, or he can buy a Beta machine that costs $87,500 and takes $163 dollars to maintain each week. How many weeks will it take for the choice of the Alpha machine to be less expensive than that of the Beta machine?

\[
y_A = 125820 + 83t \\
\]
\[
y_B = 87500 + 163t \\
\]

\[
125820 + 83t = 87500 + 163t \\
125820 - 87500 = (163 - 83)t \\
38320 = 80t \\
t = \frac{38320}{80} = 479 \text{ weeks}
\]
6. Determine the value of \( k \) for which the following system has
(a) infinitely many solutions
(b) no solution.

\[
\begin{align*}
2x + (2k - 5)y & = 3 \\
6x - 3y & = 9
\end{align*}
\]

\( \text{Note that when } k = 2, \text{ the two lines above are the same line.} \)
7. Kathy invested $73,500 in three different accounts. The annual yield on each of the three accounts was 4%, 5.5%, and 6%. The amount of money in the 4% account was four times the amount of money in the 5.5% account. If at the end of the year she had made $3,900 in interest, how much had she placed in each account?

Let
\[ x = 4\% \text{ amount} \]
\[ y = 5.5\% \text{ amount} \]
\[ z = 6\% \text{ amount} \]

\[ x + y + z = 73,500 \]
\[ x = 4y \]
\[ x - 4y = 0 \]
\[ 0.04x + 0.055y + 0.06z = 3900 \]

\[
\begin{bmatrix}
1 & 1 & 1 & 73500 \\
1 & -4 & 0 & 0 \\
0.04 & 0.055 & 0.06 & 3900
\end{bmatrix} \xrightarrow{\text{rref}}
\]

\[
\begin{bmatrix}
1 & 0 & 0 & 24000 \equiv x \\
0 & 1 & 0 & 6000 \equiv y \\
0 & 0 & 1 & 43500 \equiv z
\end{bmatrix}
\]
The Jones’ family has three birthday party celebrations this month. For Abe’s party they need 18 cupcakes, 3 bottles of juice and 12 party favors. For Ben’s party they need 21 cupcakes, 4 bottles of juice and 16 party favors. For Charlie they need 26 cupcakes, 5 bottle of juice and 20 party favors. Each cupcake costs $1.75, each bottle of juice $4.75 and each party favor $7.30. Set up two matrices whose product shows how much the Jones’ family will spend for each party as far as cupcakes, juice and party favors.

\[
\begin{bmatrix}
A & C & J & F \\
B & 18 & 3 & 12 \\
C & 21 & 16 & 16 \\
26 & 5 & 20 & \\
\end{bmatrix}
\begin{bmatrix}
(1.75) \\
(4.75) \\
(7.30) \\
\end{bmatrix}
\]

\[
\begin{bmatrix}
18*1.75+3*4.75+12*7.30 \\
21*1.75+4*4.75+16*7.30 \\
26*1.75+5*1.75+20*7.30 \\
\end{bmatrix}
\]

Answer:

Abe’s Party expen.

Ben’s Party expen.

Charlie’s party expen.
Unique Solution

\[
\begin{bmatrix}
1 & 0 & \text{number} \\
0 & 1 & \text{number} \\
0 & 0 & \text{number}
\end{bmatrix}
\]

No solution

\[
\begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 1 \\
1 & 0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}
\]
because $0 \neq 1$

\[
\begin{bmatrix}
1 & 2 & 3 & 4 & 1 \\
0 & 0 & 1 & 2 & 1 \\
0 & 0 & 0 & 1 & 1
\end{bmatrix}
\]

\[
\begin{aligned}
x + 2y + 3z + 4w &= 1 \\
z + 2w &= 1 \\
x + 2y &= 5 \\
x &= 5 - 2y
\end{aligned}
\]
Infinitely many solutions

\[
\begin{bmatrix}
1 & * & * & * \\
0 & 0 & 1 & *
\end{bmatrix}
\]

↑ free variable

\[
\begin{bmatrix}
0 & 1 & * & * \\
0 & 0 & 1 & *
\end{bmatrix}
\]

↑ free variable

\[
\begin{bmatrix}
1 & * & * & * \\
0 & 1 & * & *
\end{bmatrix}
\]

↑ free variable

Example

\[
\begin{align*}
4x_2 - 3x_3 - 5x_4 &= 7 \\
2x_2 + 0x_3 + 3x_4 &= 8
\end{align*}
\]

\[
\begin{bmatrix}
0 & 4 & -3 & -5 & | & 7 \\
1 & -2 & 0 & 3 & | & 8
\end{bmatrix}
\]

\[
\text{rref} \quad \begin{bmatrix}
1 & 0 & -3/2 & 1/2 & | & 23/2 \\
0 & 1 & -3/4 & -5/4 & | & 7/4
\end{bmatrix}
\]

\[
X_1 = \frac{3}{2} x_3 - \frac{1}{2} x_4 + \frac{23}{2} \\
X_2 = \frac{3}{4} x_3 + \frac{5}{4} x_4 + \frac{7}{4}
\]

Let \( x_3 = r \) and \( x_4 = s \)
\((\frac{3}{2} \, r - \frac{1}{2} \, s + \frac{23}{2}, \frac{3}{4} \, r + \frac{5}{4} \, s + \frac{7}{4}, \, r, \, s)\).

When 200 pencil shafts are made, the total cost is $67,600. The production cost for each pencil shaft is $13.

a) What is the fixed cost? 

\[TC = FC + (\text{cost of item}) \times (\# \text{ of items})\]

\[67,600 = FC + 13 \times 200\]

\[67,600 - 2600 = FC\]

\[65,000 = FC\]

\[\begin{cases} x_1 + 2 \, x_2 + 8 \, x_3 = 2 \\ x_1 + x_2 + 4 \, x_3 = 1 \end{cases} \quad t/s\]

\[
\begin{bmatrix}
1 & 2 & 8 & \vdots 2 \\
1 & 1 & 4 & \vdots 1
\end{bmatrix}
\xrightarrow{\text{ref}}
\begin{bmatrix}
1 & 0 & 0 & \vdots 0 \\
0 & 1 & 4 & \vdots 1
\end{bmatrix}
\]

\[x_1 = 0\]

\[\begin{pmatrix} 0, \, 1-4 \, x_3, \, x_3 \end{pmatrix} \quad x_2 + 4 \, x_3 = 1 \quad x_2 = 1 - 4 \, x_3\]

\[\text{let } x_3 = t \quad (0, \, 1-4 \, t, \, t)\]
\[
\begin{bmatrix}
1 & 8 \\
3 & 4 \\
x & -1 \\
\end{bmatrix}
- \frac{3}{2}
\begin{bmatrix}
y-1 & 1 & 8 \\
1 & 2 & 2z+1 \\
x & 8 & 1 \\
\end{bmatrix}
= 2
\begin{bmatrix}
-4 & -u \\
0 & -1 \\
5 & 7 \\
\end{bmatrix}
\]

\[
\begin{bmatrix}
1 & 8 \\
3 & 4 \\
x & -1 \\
\end{bmatrix}
+ \begin{bmatrix}
-3y+3 & -24 \\
-3 & -6 \\
-24 & -6z-3 \\
\end{bmatrix}
= \begin{bmatrix}
-8 & -2u \\
0 & -2 \\
10 & 14 \\
\end{bmatrix}
\]

\[
\begin{bmatrix}
1-3y+3 & 8-24 \\
3-3 & 4-6 \\
x-24 & -1-6z-3 \\
\end{bmatrix}
= \begin{bmatrix}
-8 & -2u \\
0 & -2 \\
10 & 14 \\
\end{bmatrix}
\]

\[
\begin{bmatrix}
4-3y = -8 \\
0 = 0 \\
x-24 = 10 \\
\end{bmatrix}
\]

\[
\begin{bmatrix}
-3y = -12 \Rightarrow y = 4 \\
16 = 2u \Rightarrow u = 8 \\
x - 24 = 10 \Rightarrow x = 34 \\
-6z - 4 = 14 \Rightarrow -6z = 14 + 4 = 18 \Rightarrow z = \frac{1}{3} \\
\end{bmatrix}
\]
In the matrix equation below, find \( a + b \)

\[
\begin{bmatrix}
7 & 2 \\
2 & 2
\end{bmatrix} \cdot 
\begin{bmatrix}
1 & 4 & -1 \\
5 & -3
\end{bmatrix} - 
\begin{bmatrix}
b \\
2
\end{bmatrix} \cdot 
\begin{bmatrix}
8 & 1 & -3
\end{bmatrix} = 
\begin{bmatrix}
-\frac{29}{3} & 11 & \frac{2}{3}
\end{bmatrix}
\]

\[
\begin{bmatrix}
7 + 4 & 28 + 10 & -7 - 6 \\
a - 8 & 4a - 20 & -a + 12
\end{bmatrix} + 
\begin{bmatrix}
-8b & -b & 3b \\
16 & -2 & + 6
\end{bmatrix} = 
\begin{bmatrix}
-29 & -33 & 2
\end{bmatrix}
\]

\[
11 - 8b = -29 \\
38 - b = 33 \checkmark \\
-13 + 3b = 2 \checkmark \\
\]

\[
\begin{align*}
\alpha - 8 & = -27 \\
4a - 20 - 2 & = -34 \checkmark \\
-a + 12 + 6 & = 21 \checkmark 
\end{align*}
\]

\[
a + b = 5 + (-3) = 2
\]

\[
\begin{bmatrix}
1 & 0 & 0 & 0 & 8 & -2
\end{bmatrix}
\] is \( \text{rref} \)?

\[
\text{No, b/c of 1}
\]