1. Urn A contains 6 white marbles and 4 red marbles. Urn B contains 3 red marbles and two white marbles. A marble is selected at random from urn A, and then placed in urn B. Finally, a marble is selected at random from urn B. What is the probability that the marble selected from urn A was red, given that the marble selected from urn B is red?

\[
P(\text{red from A} | \text{red from B}) = \frac{P(\text{red B} | \text{red A}) P(\text{red A})}{P(\text{red B})} = \frac{\frac{4}{15} \cdot \frac{3}{10}}{\frac{3}{10} + \frac{4}{15}} = \frac{4}{9} = 0.4444
\]

2. Let E and F be two events, and suppose that \(P(E)=0.35\), \(P(F)=0.55\) and \(P(E \cup F)=0.70\).
   
   (a) Compute \(P(E|F)\)

   \[
P(E|F) = \frac{P(E \cap F)}{P(F)} = \frac{0.2}{0.55} \approx 0.3636
\]

   (b) Are the events E and F independent?

   \[
P(E \cap F) \neq P(E) \times P(F)
   \]
   
   \[
   E \text{ and } F \text{ are not independent events}
   \]

\[
P(E \cup F) - P(E) + P(F) - P(E \cap F) = 0.70 = 0.35 + 0.55 - P(E \cap F)
\]

\[
P(E \cap F) = 0.35 + 0.55 - 0.70 = 0.10
\]

\[
P(E | F) = 0.90 - 0.70 = 0.20
\]
3. Suppose that E and F are independent events. If \( P(E) = 0.32 \) and \( P(E \cap F) = 0.16 \), what is \( P(F) \)?

Since \( E \) and \( F \) are independent, we know that

\[
P(E \cap F) = P(E) \cdot P(F)
\]

Usually \( P(A \cap B) = \frac{P(A \mid B) \cdot P(B)}{P(B \mid A) \cdot P(A)} \)

\[
0.16 = 0.32 \cdot P(F)
\]

\[
\frac{0.16}{0.32} = P(F) = \frac{1}{2} = 0.5
\]

4. Use the tree diagram below (to be drawn in class: 0.3, 0.5, 0.2, 0.6, 0.4) to compute the probabilities:

(a) \( P(A \cap E) \)

(b) \( P(B \cap E) \)

(c) \( P(C \cap E) \)

(d) \( P(E) \)

(e) \( P(A \mid E) \)

\[\begin{array}{c}
\text{A} \quad \frac{6}{14} \quad E \\
\text{B} \quad \frac{5}{14} \quad F \\
\text{C} \quad \frac{6}{14} \quad E \\
\text{F} \quad \frac{4}{14} \quad F \\
\end{array}\]

\[\begin{array}{l}
\text{a. } P(A \cap E) = P(E \mid A) \cdot P(A) = 0.6 \times 0.3 = 0.18 \\
\text{b. } P(B \cap E) = P(E \mid B) \cdot P(B) = 0.6 \times 0.5 = 0.3 \\
\text{c. } P(C \cap E) = P(E \mid C) \cdot P(C) = 0.6 \times 0.2 = 0.12 \\
\text{d. } P(E) = 0.6 \times 0.3 + 0.6 \times 0.5 + 0.6 \times 0.2 = 0.60 \\
\text{e. } P(A \mid E) = \frac{P(A \cap E)}{P(E)} = \frac{0.18}{0.60} = 0.3
\end{array}\]

5. Five people are selected at random.

(a) What is the probability that none of the people in this group were born on the same day of the week?

(b) What is the probability that at least two people in this group were born on the same day of the week?

\[
n(E) = \frac{6 \times 5 \times 4 \times 3}{7 \times 7 \times 7 \times 7} = \frac{30 \times 12}{7^4} \approx 0.1499
\]

\[
n(S) = 7 \times 7 \times 7 \times 7 = 7^4
\]

\[
b \quad 1 - P(E) = P(E^c) = 0.8501
\]
6. The table below gives the percentage of eligible voters grouped according to profession who responded with “voted” in the 2000 presidential election. The table also gives the percentage of people in a survey characterized by their profession.

<table>
<thead>
<tr>
<th>Profession</th>
<th>Percentage who voted</th>
<th>Percentage in each profession</th>
</tr>
</thead>
<tbody>
<tr>
<td>Professionals</td>
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<td>12</td>
</tr>
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<td>White collar</td>
<td>73</td>
<td>24</td>
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<td>66</td>
<td>32</td>
</tr>
<tr>
<td>Unskilled</td>
<td>57</td>
<td>10</td>
</tr>
<tr>
<td>Farmers</td>
<td>68</td>
<td>8</td>
</tr>
<tr>
<td>Housewives</td>
<td>66</td>
<td>14</td>
</tr>
</tbody>
</table>

If an eligible voter who participated in the survey and voted in the election is selected at random, what is the probability that this person is a housewife?

7. The probability distribution of a random variable $X$ is shown in the table below.

<table>
<thead>
<tr>
<th>$X$</th>
<th>$-5$</th>
<th>$-3$</th>
<th>$-2$</th>
<th>$0$</th>
<th>$2$</th>
<th>$3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P(X=x)$</td>
<td>.17</td>
<td>.13</td>
<td>.33</td>
<td>.16</td>
<td>.11</td>
<td>.10</td>
</tr>
</tbody>
</table>

Compute:
(a) $P(X \leq 0)$
(b) $P(X \leq -3)$
(c) $P(-2 \leq X \leq 2)$
(d) $P(X = -2)$
(e) $P(X > 0)$
(f) $P(X = 1)$

\[ a. \ P(X < 0) = P(X = -5) + P(X = -3) + P(X = -2) + P(X = 0) = 0.17 + 0.13 + 0.33 + 0.16 = 0.79 \]
\[ b. \ P(X \leq -3) = 0.17 + 0.13 = 0.30 \]
\[ c. \ P(-2 \leq X \leq 2) = P(X = -2) + P(X = 0) + P(X = 2) = 0.33 + 0.16 + 0.11 = 0.60 \]
\[ d. \ P(0 < X < 2) = P(X = 2) + P(X = 3) = 0.21 \]
\[ e. \ P(X > 0) = P(X = 2) + P(X = 3) = 0.21 \]
\[ f. \ P(X = 1) = 0.10 \]
8. The management of a big company has projected the sales of its products (in millions of dollars) for the upcoming year, with the associated probabilities shown in the following table:

<table>
<thead>
<tr>
<th>Sales</th>
<th>20</th>
<th>22</th>
<th>24</th>
<th>26</th>
<th>28</th>
<th>30</th>
</tr>
</thead>
<tbody>
<tr>
<td>Probability</td>
<td>.05</td>
<td>.10</td>
<td>.35</td>
<td>.30</td>
<td>.15</td>
<td>.05</td>
</tr>
</tbody>
</table>

(a) What does management expect the sales to be next year?
(b) Compute the variance of the random variable from the above table.
(c) Compute the standard deviation.
(d) Compute the mode.

\[ E(Sales) = 20 \times .05 + 22 \times .10 + 24 \times .35 + 26 \times .30 + 28 \times .15 + 30 \times .05 \]
The management of a big company has projected the sales of its products (in millions of dollars) for the upcoming year, with the associated probabilities shown in the following table:

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</thead>
<tbody>
<tr>
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<td>.05</td>
<td>.10</td>
<td>.35</td>
<td>.30</td>
<td>.05</td>
<td></td>
</tr>
</tbody>
</table>

(e) What does management expect the sales to be next year?
(f) Compute the variance of the random variable from the above table.
(g) Compute the standard deviation.
(h) Compute the mode. 24.

9. A fair die is rolled five times. Calculate the probabilities of obtaining:
   (a) Exactly two sixes
   (b) At least two sixes
   (c) At most two sixes
   (d) Between two and four sixes, included.

   a. $\text{binompdf}(5, \frac{1}{6}, 2) \approx 0.1608$
   b. $P(X \geq 2) = 1 - P(X \leq 1) = 1 - \text{binomcdf}(5,\frac{1}{6}, 1) \approx 0.1962$
   c. $P(X \leq 2) = \text{binomcdf}(5, \frac{1}{6}, 2) \approx 0.9645$
   d. $P(2 \leq X \leq 4) = \text{binomcdf}(5, \frac{1}{6}, 4) - \text{binomcdf}(5,\frac{1}{6}, 1) \approx 0.1961$
9. A player pays $3 to play the following game: he tosses three fair coins and receives back "payoffs" of $1 if he tosses no heads, $2 for one head, $3 for two heads, and $4 for three heads. Find the player's expected net winnings for this game.

<table>
<thead>
<tr>
<th>Outcomes</th>
<th>Net Win $</th>
<th>Prob</th>
<th>Expected net win:</th>
</tr>
</thead>
<tbody>
<tr>
<td>HTA</td>
<td>4-3-3</td>
<td>1/8</td>
<td>( \frac{1}{8} \cdot 1 + \frac{1}{8} \cdot 0 + \frac{1}{8} \cdot 0 + \frac{1}{8} \cdot 0 + \frac{1}{8} \cdot (-1) + \frac{1}{8} \cdot (-1) + \frac{1}{8} \cdot (-1) + \frac{1}{8} \cdot (-2) = )</td>
</tr>
<tr>
<td>THT</td>
<td>2-3-3</td>
<td>1/8</td>
<td>( \frac{1}{8} - \frac{3}{8} - \frac{2}{8} = -\frac{4}{8} = \frac{5}{8} )</td>
</tr>
<tr>
<td>TTH</td>
<td>2-3-3</td>
<td>1/8</td>
<td>( \frac{1}{8} - \frac{3}{8} - \frac{2}{8} = -\frac{4}{8} = \frac{5}{8} )</td>
</tr>
<tr>
<td>TTT</td>
<td>1-3-2</td>
<td>1/8</td>
<td>( \frac{1}{8} - \frac{3}{8} - \frac{2}{8} = -\frac{4}{8} = \frac{5}{8} )</td>
</tr>
</tbody>
</table>

10. The $3 cost to play the game of Example 2 makes the game unfair against the player (since the player's expected net winnings are negative). What cost would make this a fair game?

Let \( A \) be the "fair" amount to pay.

\[
\frac{1}{8}(4-A) + 3 \cdot \frac{1}{8}(3-A) + 3 \cdot \frac{1}{8}(2-A) + \frac{1}{8}(1-A) = 0
\]

\[
\frac{4-A}{8} + \frac{9}{8} - \frac{3A}{8} + \frac{6}{8} - \frac{3A}{8} + \frac{1}{8} - \frac{A}{8} = 0
\]

\[
\frac{4+9+6+1}{8} = (\frac{1+3+3+1}{8}) A
\]

\[
\frac{20}{8} = \frac{8}{8} A \Rightarrow 20 = 8A \Rightarrow A = \frac{20}{8} = 2.5
\]

If he pays $2.50, the game is fair.
10. A box has 3 yellow, 7 red, and 4 black balls. Three balls are drawn at the same time (i.e. without replacement) from the box. Let $X$ be the number of red balls drawn.

a) Compute $P(X=2)$

$$P(X=2) = \frac{n(E)}{n(S)} = \frac{C(7,2)C(7,1)}{C(14,3)} = \frac{4038}{4533} \approx 0.89038$$

b) Compute the probability of selecting exactly one red ball or two black balls

$$P(1r \cup 2b) = P(1r) + P(2b) - P(1r \cap 2b) = \frac{C(7,1)C(7,2) + C(4,2)C(10,1) - C(7,3)C(4,2)}{C(14,3)}$$

$$= \frac{= 0.4533}{4533}$$

11. A man wishes to purchase a 5-year term-life insurance policy that will pay the beneficiary $20,000 in the event that the man’s death occurs during the next 5 years. Using life insurance tables, he determines that the probability that he will live another 5 years is 0.96. What is the minimum amount that he can expect to pay for his premium? Hint: The minimum premium occurs when the insurance company’s expected profit is zero.

<table>
<thead>
<tr>
<th>Outcomes</th>
<th>Gain $</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>Man lives</td>
<td>$P</td>
<td>0.96</td>
</tr>
<tr>
<td>Man dies with $P - 20,000</td>
<td>0.04</td>
<td></td>
</tr>
</tbody>
</table>

Set up expected gain $\geq 0$

$$0.96P + 0.04(P-20,000) \geq 0$$

$$0.96P + 0.04P - 0.04 \times 20,000 \geq 0$$

$$1P \geq 0.04 \times 20,000 = 800$$

$$\text{Minimum premium} = \text{ } 800$$
12. A woman purchased a $20,000, 1-year term life insurance policy for $260. Assuming that the probability that she will live for another year is .992, find the company’s expected gain.

\[
\begin{array}{|c|c|c|}
\hline
\text{Outcomes} & \text{Gain} & \text{Prob} \\
\hline
\text{Lives 1 yr} & 260 & .992 \\
\text{Dies in 1 yr} & 260-20000 & .008 \\
\hline
\end{array}
\]

\[
\text{Expected gain} = .992 \times 260 + .008 \times (20000-260) = 256.76 = 100.00
\]

13. The monthly gas bill (in dollars) for a typical family from May 2009 to April 2010 is summarized in the following table.

<table>
<thead>
<tr>
<th>Month</th>
<th>MAY</th>
<th>JUN</th>
<th>JUL</th>
<th>AUG</th>
<th>SEP</th>
<th>OCT</th>
<th>NOV</th>
<th>DEC</th>
<th>JAN</th>
<th>FEB</th>
<th>MAR</th>
<th>APR</th>
</tr>
</thead>
<tbody>
<tr>
<td>Amount</td>
<td>94.57</td>
<td>55.99</td>
<td>41.15</td>
<td>36.15</td>
<td>48.64</td>
<td>74.97</td>
<td>142.02</td>
<td>195.46</td>
<td>237.97</td>
<td>238.2</td>
<td>219.14</td>
<td>170.31</td>
</tr>
</tbody>
</table>

(a) Compute the mean of the amount of the gas bill for a typical family, for the 12-month period under consideration.

\[\text{mean} = \$129.46\]

(b) Compute the standard deviation for the data (round to four decimal places).

\[\text{ST. DEV.}.765\]

\[\text{Variance}: 5852.47\]

14. An algebra quiz consists of eight multiple choice questions. Six must be answered correctly to receive a passing grade. If each question has five possible answers, of which only one is correct, what is the probability that a student who guesses at random on each question will pass the examination?

Binomial
\[p(\text{success}) = \frac{1}{5}\]
\[n = \# \text{ of trials} = 8\]
\[r = 6\]

\[
\begin{align*}
\text{Let } X &= \# \text{ of correct answers (successes)} \\
P(X \geq 6) &= P(6 \leq X \leq 8)
\end{align*}
\]
15. An automobile-manufacturing company uses ten industrial robots as welders on its assembly line. On a given working day, the probability that a robot will be inoperative is .05. What is the probability that on a given working day:
(a) Exactly two robots are inoperative?
(b) More than two robots are inoperative?

\[
a. \quad \text{binom pdf}(10, .05, 2) \approx .0746 \\
b. \quad 1 - \text{binom cdf}(10, .05, 2) \approx .0115
\]

16. Two cards are drawn without replacement from a well-shuffled deck of 52 playing cards. What is the probability that the second card drawn is a diamond?

\[
P(\text{2nd card Diamond}) = \frac{4}{52} \cdot \frac{13}{51} + \frac{39}{52} \cdot \frac{13}{51} \cdot \frac{3}{50} \\
= \frac{1}{17} + \frac{39}{204} = .25
\]
17. Five cards are dealt at random without replacement from a well-shuffled deck of 52 playing cards. What is the probability of dealing
(a) “four of a kind” (i.e. four of the same rank and any other card)?
(b) “two pairs” (i.e. 2 cards of the same rank and two cards of any other rank, with an unmatched card)?

$$a. \frac{13 \cdot \binom{4}{4} \binom{48}{1}}{\binom{52}{5}} \approx 0.0002$$

$$b. \frac{\binom{13}{2} \binom{4}{2} \binom{44}{1}}{\binom{52}{5}} \approx 0.0079$$
1. The table below gives the percentage of eligible voters grouped according to profession who responded with “voted” in the 2000 presidential election. The table also gives the percentage of people in a survey characterized by their profession.

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If an eligible voter who participated in the survey and voted in the election is selected at random, what is the probability that this person is a housewife?