Math 151 WIR 4: Exam 1 Review

1. If \( \cot(x) = -\frac{14}{13} \) and \( \cos x < 0 \), what is the exact value of \( \sin(2x) \)?

2. Solve the equation \( 2 \sin^2 x - \sin x - 1 = 0 \), \( 0 \leq x \leq 2\pi \).

3. Find a vector that has opposite direction to \( \langle -2, 4 \rangle \) and length 6.

4. Consider the vectors \( \mathbf{a}=\langle 4, 3 \rangle \), \( \mathbf{b}=\langle 3, -1 \rangle \), \( \mathbf{c}=\langle 9, 4 \rangle \). Find the exact values of \( s \) and \( t \) such that \( \mathbf{c} = s \mathbf{a} + t \mathbf{b} \).

5. Suppose that a wind is blowing in the direction S45°E at a speed of 60 km/h. A pilot is steering a plane in the direction N60°E with airspeed (speed in still air) of 300 km/h. The true course, or track, of the plane is the direction of the resultant of the velocity vectors of the plane and the wind. The ground speed of the plane is the magnitude of the resultant. Find the true course and the ground speed of the plane.

6. Find the cosine of angle \( \hat{A}\hat{C}\hat{B} \), where \( A(2, 0) \), \( B(5, 6) \), \( C(-1, 4) \).

7. Find the distance from point \( (3, 1) \) to the line \( y = 2x + 1 \).

8. Which of the following best describes the curve generated by the parametric equations
   a. \( x = 3 + sint, \ y = cost - 1 \), as \( 0 \leq t \leq 2\pi \)
   b. \( x = \frac{1}{t}, \ y = \frac{3}{t^2} - 1 \), as \( 3 \leq t \leq 6 \)
   c. \( x = 3sint - 2, \ y = cost + 1 \), as \( 0 \leq t \leq 2\pi \)
   (i) an arc of a circle; (ii) part of a hyperbola; (iii) a straight line segment;
   (iv) part of an ellipse; (v) part of a parabola; (vi) not enough information to decide.

9. Find parametric equations for the line that passes through the point \( (-1, 7) \) and is parallel to the line \( 2x - 5y + 1 = 0 \). Then find parametric equations for the line that passes through the point \( (-1, 7) \) and is orthogonal (perpendicular) to the line \( 2x - 5y + 1 = 0 \).

10. Determine the value of the limit: \( \lim_{x \to -2} \frac{x + 3}{x + 2} \)

11. Determine the value of the limit: \( \lim_{x \to 6} \frac{7 - x}{(x - 6)^2} \)

12. Evaluate the limit: \( \lim_{t \to 9^+} \frac{9 - t}{3 - \sqrt{t}} \)

13. Evaluate the limit: \( \lim_{t \to 0} \left( \frac{1}{t} - \frac{1}{t^2 + t} \right) \)

14. Evaluate the limit: \( \lim_{x \to 5} \frac{x^2 - 9x + 18}{x - 5} \)
15. Let \( g(x) = \frac{x^2 + x - 12}{|2x-6|} \). Comment on the continuity of \( g \).

16. Find the value(s) of \( c \), if any, that make \( f \) continuous everywhere.

\[
f(x) = \begin{cases} 
(x + c)^2 & x < 3 \\
5x + c & x \geq 3 
\end{cases}
\]

17. Use the Intermediate Value Theorem to show that there are two real numbers, one positive and one negative, which are roots of the equation \( x^4 + x = 8 \).

18. Find the limit: \( \lim_{x \to \infty} \frac{1}{5x-4} \)

19. Find the limit: \( \lim_{t \to \infty} \frac{3t^4 + 1}{(t^3-1)(2t+1)} \)

20. Find the limit: \( \lim_{u \to -\infty} \frac{u+7}{\sqrt{9u^2+1}} \)

21. Find the limit: \( \lim_{x \to -\infty} (x + \sqrt{x^2 - 3x}) \)

22. Find the limit: \( \lim_{x \to \infty} (\sqrt{4x^2 - 3x - 2x}) \)

23. Find the horizontal and vertical asymptotes of the curve \( y = \frac{-6x^2 + x - 8}{x^2 + x - 42} \).

24. Find the vertical asymptotes of the graph of \( f(x) = \frac{-4x^3 - x^2 + 3x}{3x - 4x^2} \). Are there any horizontal asymptotes?

25. Find the slope of the tangent line to the curve at the point \((-1, \frac{1}{\sqrt{6}})\): \( y = \frac{1}{\sqrt{1-5x}} \).

26. Find \( f'(x) \) from the definition of derivative, if \( f(x) = x^3 - x^2 - x + 2 \).