Modular Categories and Applications
Abstracts
as of:

March 14, 2009

- **Alexei Davydov** (Macquarie) *Witt group of modular categories and Moore-Seiberg conjecture*

  Moore and Seiberg conjectrured that all Rational Conformal Field Theories can be obtained from Wess-Zumino-Witten theories. We reformulate this conjecture in terms of possible generators of the Witt group of (unitary) modular categories. We also make a few observations about possible relations.

- **Terry Gannon** (Alberta)

- **Yi-Zhi Huang** (Rutgers) *Nonsemisimple braided tensor categories and logarithmic conformal field theories*

  The logarithmic operator product expansion was first studied in physics by Gurarie. The underlying logarithmic conformal field theories have been developed rapidly in recent years by both physicists and mathematicians. In this talk, I will discuss the logarithmic tensor product theory for modules for a vertex operator algebra developed by Lepowsky, Zhang and myself and its connection with logarithmic conformal field theories. In the case that the vertex operator algebra satisfies natural positive-energy and cofiniteness conditions, I have proved that the conditions to use the logarithmic tensor product theory are satisfied. In particular, the category of modules for such a vertex operator algebra is a (not-necessarily-semisimple) braided tensor category and the logarithmic operator product expansion exists for the corresponding logarithmic conformal field theories.

- **Joanna Kania-Bartoszynska** (NSF) *Topological applications of quantum invariants.*

  I will discuss several applications of quantum invariants of 3-manifolds derived form topological quantum field theories. Examples include criteria for periodicity of knots and manifolds, obstructions to embedding one manifold into another, and combinatorial formulas for computing integrals of simple closed curves over character variety of the surface against Goldman's symplectic measure.

- **Alexei Kitaev** (Caltech) *Bose-condensation and edges of topological quantum phases.*

  Various physical phenomena pertaining to topological quantum phases are given a categorical interpretation, which leads to a set of plausible mathematical conjectures. If a two-dimensional phase is characterized by a unitary modular category $\mathcal{M}$, then possible condensates of anyonic particles are given by commutative Frobenius algebras. A "hard" (fully gapped) edge is described by the module category over a commutative Frobenius algebra of maximal dimension. Conversely, if the edge is given by a unitary tensor category $\mathcal{C}$, then the bulk is its center: $\mathcal{M} = Z_1(\mathcal{C})$. The consideration of a corner between two edges suggests that two unitary tensor categories are Morita equivalent if and only if their centers are isomorphic. I will discuss a few more relations of this kind.

- **Scott Morrison** (Microsoft) *Discovering knot polynomial identities using the D_{2n} planar algebras.*

  I'll tell you about the D_{2n} planar algebras, which are Z/2Z quotients of the Temperley-Lieb planar algebras. These planar algebras aren't braided, but have a braiding on the 'even part'. This lets us define some knot invariants, which we can recognise as being related to two different quantum knot polynomials. This provides a new source of identities between knot polynomials. I'll also explain these identities in terms of isomorphisms between certain small modular tensor categories. These isomorphisms come from three sources: Kirby-Melvin symmetry, triality for SO(8), and a degenerate case of level-rank duality for SO(3)-SO(k).

- **Michael Mueger** (Nijmegen) *Embedding pre-modular categories into modular ones.*

  A number of years ago I formulated a conjecture on embeddings of pre-modular categories (=braided fusion categories) into modular categories of minimal size (=dimension). I will show that this conjecture, restricted to pre-modular categories with even symmetric center, is equivalent to another, intuitively more accessible, conjecture. The latter concerns crossed products of modular categories by finite group actions. I'll also discuss a tentative approach to proving that conjecture.

- **Richard Ng** (Iowa State) *Modular Tensor Categories and Congruence Subgroups of SL(2,Z)*

  Associated to each modular tensor category is a natural projective representation of the modular group SL(2,Z). The kernel of this projective representation has been shown to be a congruence subgroup of SL(2,Z) and its level is equal to the order of the ribbon structure. This result is a consequence of an action of SL(2,Z) on a set of functionals called generalized Frobenius-Schur indicators. In this talk, we will discuss the definition of generalized Frobenius-Schur indicators and their application to this result.

The talk is part of joint work with Peter Schauenburg.

- **Dmitri Nikshych** (New Hampshire) *The core a braided fusion category*

  We introduce a new notion of the core of a braided fusion category C. It is defined in terms of the de-equivariantization of the centralizer of a maximal Tannakian subcategory of C (following the construction of Bruguieres and Muger). We will show that the core is an invariant of C and describe its structure. It allows to separate the part of a braided fusion category that does not come from finite groups. We will also discuss the relation between the core of the center of a fusion category and its Morita equivalence class. This talk is a report on joint works with V. Drinfeld, P. Etingof, S. Gelaki, and V. Ostrik.

- **Victor Ostrik** (Oregon) *Witt group of non-degenerate braided fusion categories.*

  I will talk on my joint work with M.Mueger and D.Nikshych. We consider a monoid formed by equivalence classes of non-degenerate braided fusion categories (where non-degeneracy is a condition equivalent to modularity in the presence of spherical structure) with operation induced by external tensor product. The Witt group is by definition a quotient of this monoid by the relation which says that class of a Drinfeld center is zero. I will explain basic results about this group and its relation to a classical Witt group of quadratic forms on finite abelian groups.

- **Nick Read** (Yale, Physics)

- **Joost Slingerland** (Dublin ITP) *Phase transitions and domain walls in 2+1 dimensional topological field theory*

  Recently there has been much interest in 2+1 dimensional physical systems with "topological order". At low energies, the phases of such systems can be described by topological field theory, in particular their excitations may have nontrivial braiding and fusion interactions described by a suitable unitary braided tensor category. Using a knowledge of just this fusion and braiding as a starting point, one may ask whether it is still possible to make useful statements about phase transitions that may occur. I will argue that this is indeed the case for phase transitions caused by an analogue of Bose condensation, and indicate how one may obtain the spectrum, fusion and braiding of the condensed phase. In particular this gives a way of constructing new TQFTs from TQFTs that have bosonic objects. I will also explore relations to similar constructions known in Conformal Field Theory, notably the coset construction.

- **Vladimir Turaev** (Indiana)

- **Hans Wenzl** (UC San Diego) *On tensor categories of spinor type*

  Let U be the Drinfeld-Jimbo deformation of the universal enveloping algebra of an orthogonal Lie algebra, and let S be its spinor representation. We show that the U-intertwiners of its m-th tensor power can be described in terms of a q-deformation of

$U\mathfrak{so}_m$ which is different from the corresponding quantum group. We discuss applications to reconstructing tensor categories with the fusion ring of spinor groups.

- **Feng Xu** (UC Riverside) *General constructions of rational nets and examples of modular tensor categories*

  A large class of modular tensor categories come from representations of rational nets. In this talk I will discuss some general constructions of rational nets such as cosets, orbifolds and more recently mirror extensions using ideas from operator algebras, and examples of modular tensor categories from these constructions.

- **Shigeru Yamagami** (Ibaraki) *Multicategories of planar diagrams and tensor categories*

  Geometric or diagrammatic thinking enables us to manipulate algebraic relations in an intuitive and efficient way. Lots of quantum invariants are introduced and investigated with the help of such machinery.

  I here focus on planar diagrams, which are one-dimensional geometric objects (strings) in a plane. A basic example is the ones introduced by Kauffman to describe the Temperley-Lieb algebras, where the rigidity of the accompanied tensor category is encoded in the planar isotopy of strings. This view point, i.e., the interpretation of rigidity (or pivotality) on tensor categories as planar isotopy, is fairly universal as observed by Freyd and Yetter, and has been witnessed through several independent resources such as representationes of quantum groups, symmetry in solvable models of statistical mechanics and so on.

  Related to subfactor theory in operator algebras, the rigidity is the heart of the subject because it is equivalent to the finiteness of Jones indices and the source for interesting combinatorial structures, known as towers of algebras (V.Jones), standard invariants (S.Popa) or paragroups (A.Ocneanu). The relevant information is lately paraphrased by Jones as the notion of planar algebras, which appears together with planar diagrams fulfilling extra conditions. Other than the original definition, planar algebra allows various related versions in a more or less obvious way In this talk, I shall review such variants and clarify how they are connected with pivotal (bi)categories. Planar presentation is especially useful in describing free products of tensor categories as investigated by Bisch and Jones, and some of related results will be also discussed.