Instructions:

(1) Consider the function \( f(x) = \sqrt{x} \). (5 points)

(a) Find a linear approximation to \( f \) at \( x = 9 \).
(b) Use part (a) to approximate \( \sqrt{7} \).

(2) Find all local maxima and minima of: (15 points)

\[ f(x) = x^2 e^x - 3e^x \]
(3) Evaluate: \[ \int_{1}^{2} x^{3/2} \, dx \] (5 points)

(4) Evaluate: \[ \int_{0}^{1} x^3 \sqrt{x^4 + 3} \, dx \] (5 points)

(5) Evaluate: \[ \int_{1}^{2} \frac{(\ln x)^3}{x} \, dx \] (5 points)
(6) Consider the following function: 

\[ f(x,y) = x^2 y + 2y^3 + 5y^2 + x^2 \]

(a) Compute all partial derivatives of \( f \): 
\( f_x, f_y, f_{xx}, f_{yy}, \) and \( f_{xy} \).

(b) Find all critical points of \( f \). \textit{Hint: there are 4.}

(c) Classify each critical point from (b) as local minima, maxima or saddle points.

(7) Find the maximum and minimum of the function

\[ f(x,y) = x^2 + y \] subject to the constraint:

\[ g(x,y) = x^4 + y^2 = 2 \]

\textit{Hint: there are 4 solutions to the Lagrangian system.}
(8) Check if the functions below satisfy the differential equation: (10 points)

\[ xy' = y \]

(a) \( y = x \ln x \)

(b) \( y = x \)

(9) (20 points)

Newton’s Law of cooling states that the temperature, \( H \), of an object changes at a rate proportional to the difference between \( H \) and the air temperature, with some proportion constant \( k \).

(a) Write a differential equation describing the temperature \( H \) of a cold beverage over time \( t \) if the air temperature is 100°F.

(b) Suppose at time \( t = 0 \) the temperature of the beverage is 35°F and after 30 minutes (\( t = 30 \)) the beverage is 70°F. Find the particular solution to the differential equation in part (a) with these conditions.

(c) What was the temperature of the beverage at \( t = 15 \)?

(d) At what time is the temperature of the beverage 90°F?
(e.c.) Evaluate: (Extra Credit 4 points)

(a) \[ \int xe^{-3x} \, dx \]

(b) \[ \int \frac{x + 1}{x(x - 1)} \, dx \]