(1) Evaluate the following limits: (20 points)
(a) \( \lim_{x \to 0} \frac{\sqrt{3 + x} - \sqrt{3}}{x} \)
(b) \( \lim_{x \to \infty} \sqrt{9x^2 + x - 3} \)
(c) \( \lim_{x \to 0} \frac{3x^2 - 7x + 1}{x^2 - 1} \)
(d) \( \lim_{x \to \pi} a^2 - b \)

(2) Find a value of \( a \) so that the following limit exists. Then evaluate the limit. (15 points)
\[
\lim_{x \to -2} \frac{3x^2 + ax + a + 3}{x^2 + x - 2}
\]
EXAM

(3) Hard Problems! (15 points)

(a) Use the Squeeze Theorem to prove that
\[ \lim_{x \to 0} x^2 e^{\cos \left( \frac{x}{2} \right)} = 0 \]

(b) Use the Intermediate Value Theorem to prove that
\[ e^x - 3x \]
has a root in each of the intervals: \([-1, 0]\) and \([0, 1]\).
(4) Find, for the following rational function: 
\[ f(x) = \frac{(3x - 2)^2(x + 2)(x - 1)}{(3x - 2)(x + 2)^2(4x - 1)} \]

(a) All zeros

(b) All removable singularities

(c) All vertical asymptotes

(d) The horizontal asymptote

(5) Compute the derivative of \( g \) using the limit definition: 
\[ g(x) = \frac{x^2 + 1}{x - 2} \]

at \( x = a \) for full points, or at \( x = 0 \) for 8 points.
Consider the following function: (15 points)

\[
h(x) = \begin{cases} 
  e^x & x < 0 \\
  -\frac{3\sqrt{x+1}}{x-3} & 0 \leq x < 3 \\
  x^2 & 3 \leq x 
\end{cases}
\]

Find all discontinuities (singularities) of \( h \) and evaluate the left- and right-hand limits at each.