PRACTICE EXAM III M211

Instructions: Give exact answers when possible.

(1) Consider the function: (30 points)

\[ f(x) = 2x^2 \ln(x) - x^2 \]

(a) Find all local maxima and minima for \( f \)

(b) Find all intervals on which \( f \) is concave up and concave down.
(2) Evaluate the following limits (using L’Hospital’s Rule): (20 points)

(a) \( \lim_{x \to 0} \frac{\cos(x) - 1}{x^2} \)

(b) \( \lim_{x \to \infty} (\ln(x))^{1/\ln(x)} \)

(3) Harder Problem! (10 points)
Use the Mean Value Theorem to show that if \( f \) and \( g \) are continuous on \([a, b]\) and \( f'(x) = g'(x) \) on \([a, b]\) then for some constant \( k \), \( f(x) - g(x) = k \) for all \( x \in [a, b] \).
(4) (20 points)

Find the point on the line $y = mx + b$ that is closest to the origin.

(5) (20 points)

A particle is moving along a straight line with acceleration $a(t) = 2t + 1 \text{ ft/s}^2$
where $t$ is time in seconds. Find an equation describing the displacement if the
velocity at $t = 0$ is 1 ft/s and the displacement at $t = 0$ is 2.