Math 220 Final Exam Practice Problems
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The following are just a few representative problems. They are not meant to include examples of all possible problems that may be on the exam. You should also be prepared to work any problems similar to homework, examples from class, and the first two exams.

1. Prove or find a counterexample:

(a) For all real numbers x, x is irrational if, and only if, $10x$ is irrational.

(b) For all real numbers x, x is irrational if, and only if, $\sqrt{2}x$ is irrational.

2. Prove that $\displaystyle\sum_{i=1}^{n} i(i+2) = \frac{n(n+1)(2n+7)}{6}$ for all natural numbers n.

3. Let A, B, and C be sets contained in a universal set. Prove that

$$(A - B) - C = A - (B \cup C).$$

4. Let $f : \mathbb{R} \to \mathbb{R}$ be the function defined by $f(x) = x^2 + 1$.
(a) Find $f^{-1}((0, 2))$ (where $(0, 2)$ denotes an open interval).

(b) Find the range of f.

(c) Is f one-to-one? Justify your answer.

5. Let A and B be sets, and let $f : A \to B$ be a function. Let X be a subset of A, and let Y be a subset of B for which $f(X) \subseteq Y$.
(a) Prove that $X \subseteq f^{-1}(Y)$.

(b) If $f(X) = Y$, is it necessarily true that $X = f^{-1}(Y)$? Justify your answer.

6. Find a solution to the equation $14x + 18y = 114$ in which x and y are integers.

7. Let $A = \{1, 2, 3\}$ and let X be the set of all bijective functions $f : A \to A$. Define a relation R on X by fRg if and only if $f(1) = g(1)$.
(a) Prove that R is an equivalence relation.

(b) Let $n = 4$. Find all elements in the equivalence class of the function f defined by $f(1) = 2$, $f(2) = 3$, and $f(3) = 1$.

8. Prove that if n is an integer for which $5 \nmid n$, then $n^2 \equiv 1 \mod 5$ or $n^2 \equiv 4 \mod 5$.

9. Let $\mathbb{Z}_9$ be the set of congruence classes of integers modulo 9. Find the subset of $\mathbb{Z}_9^*$ consisting of all elements $[a]_9$ for which there exists $[x]_9 \in \mathbb{Z}_9$ such that $[a]_9 \cdot_9 [x]_9 = [0]_9$.

10. Let $X = \{a, b, c, d, e, f\}$. Which of the following are partitions of X?

(a) $\{\{e, f\},\ \{a, c, f\},\ \{b, d\}\}$

(b) $\{\{e, f\},\ \{a, c\}, \{b, d\}\}$

(c) $\{\{a, b, c, d\},\ \{e\},\ \{f\}\}$

(d) $\{\{a, c, d\},\ \{b, f\}\}$