**Math 220     Practice for Exam 1**

1. Consider the statement: For all integers $m$ and $n$, if $m$ is even and $n$ is even , then $m + n$ is divisible by 4.

(a) Write the converse of this statement.

(b) Write the contrapositive of this statement.

(c) Write the negation of this statement.

(d) [5] Which of the above four statements *(the proposition, its converse (a), its contrapositive (b), its negation (c))* are true? (You need not justify your answer.)

2. Consider the statement: For all real numbers $x$ and $y$, if $xy$ is rational, then $x$ is rational.

(a) Write the converse of this statement.

(b) Write the contrapositive of this statement.

(c) Write the negation of this statement.

(d) Which of the above four statements *(the proposition, its converse (a), its contrapositive (b), its negation (c))* are true? (You need not justify your answer.)

3. Prove that for all integers $m$ and $n$, if $m$ and $n$ are both odd, then $m + n$ is even. Is the converse true?

4. Prove that for all integers $n$, $n$ is divisible by 3 if, and only if, $n^2$ is divisible by 3.

5. Prove there do not exist integers $m$ and $n$ for which $9m + 51n = 2$.

6. Prove by induction that for each positive integer $n$,
$$2 + 6 + 10 + \cdots + (4n + 2) = 2(n + 1)^2.$$

7. Prove by induction that for all integers $n \geq 1$, 3 divides $n^3 + 2n$.