Math 365-501 Exam 3
April 17, 2009
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Name____

There are 9 questions, for a total of 100 points. Point values are written beside each question. *No calculators allowed. Show your work for full credit.*

1. [11 points] In an arithmetic sequence, the fourth term minus the first term equals 12. The sum of the first and the fourth term is -8. Find the eighth term of the sequence.

2. [9] Find a digit to fill in the blank, if possible, so that the number

$$6154\,\underline{\quad}\,2$$

is divisible by

(a) 3

(b) 4

(c) 11

3. (a) [10] Find the GCD for 350 and 588 using the Euclidean algorithm.

(b) [5] Find the LCM for 350 and 588.

(c) [5] Find the GCD and LCM for 12! and 13!.

4. [10] Paper plates come in packages of 20, paper cups in packages of 8, and napkins in packages of 16. What is the least number of plates, cups, and napkins that can be purchased so that there is an equal number of each?

5. [6] Fill in each of the blanks so that that answer is nonnegative and the least possible number:

(a) $165473 \equiv$ ____________(mod 5)

(b) $265473 \equiv$ ____________(mod 9)

6. [10] If a fraction is equal to $\frac{2}{3}$, and the sum of the numerator and denominator is 20, what is the fraction?

7. [10] Mariah added fractions on her paper as follows:

$$\frac{1}{2} + \frac{1}{3} =$$

Do the addition correctly, and draw a picture that will help Mariah understand.

8. [12] Find the simplest form for each of the following:

(a) $\left(\frac{2}{5}\right)^2 + (-2)^4 \div 5 \cdot \frac{1}{2} + 5^{-2}$

(b) $\dfrac{x^2 - y^2}{x^2 + xy}$

9. [12] (**True/False/Counterexample**) For each statement, indicate whether it is true (T) or false (F). If it is false, give a counterexample.

(a) For all integers a, b, and c, if $ac = bc$, then $a = b$.

(b) For all integers a and b, $|a + b| = |a| + |b|$.

(c) For all integers a, b, and c, if $c|a$ and $c|b$, then $c|(a + b)$.

(d) For all integers a and b, if a and b are both even, then $\text{GCD}(a, b) = 2$.

(e) For all natural numbers a, b, and c, $\dfrac{a+b}{a+c} = \dfrac{b}{c}$.

(f) For all natural numbers a, b, and c, $\dfrac{ab+bc}{b} = a + c$.