Math 365 Exam 3
November 19, 2010
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Name_____

There are 8 questions, for a total of 100 points. Point values are written beside each question. *No calculators allowed. Show your work for full credit.*

1. [10] **Short answer.** For each, answer the following question, filling in the blank with "yes" or "no". Is the number 236,808 divisible by

2 _____ 3 _____ 4 _____ 5 _____ 6 _____

7 _____ 8 _____ 9 _____ 10 _____ 11 _____

2. (a) [9] Find the GCD for 510 and 690 using the Euclidean algorithm.

(b) [5] Find the LCM for 510 and 690.

(c) [5] Do any primes less than 23 divide the number $2 \cdot 3 \cdot 5 \cdot 7 + 11 \cdot 13 \cdot 17 \cdot 19$?

3. [10] Hot dogs come in packages of 10, buns in packages of 8, and paper plates in packages of 30. What is the least number of hot dogs, buns, and plates that can be purchased so that there is an equal number of each?

4. [10] Fill in each of the blanks so that the answer is nonnegative and the least possible number:

(a) $27,457 \equiv$ _____(mod 3)

(b) $27,457 \equiv$ _____(mod 11)

5. [10] If a fraction is equal to $\frac{1}{6}$, and the sum of the numerator and denominator is 28, what is the fraction?

6. [12] Find the simplest form for each of the following:

(a) $\left(\frac{1}{3}\right)^3 + (-1)^4 \div 9 \cdot \frac{3}{2} + 3^{-2}$

(b) $\frac{a^2 - b^2}{ab - b^2}$

7. [8] Which of the following represent terminating decimals? Circle all those that do.

$\frac{3}{2}$ $\frac{7}{3}$ $\frac{3}{5}$ $\frac{3}{20}$ $\frac{18}{72}$ $\frac{10}{57}$ $\frac{1}{140}$ $\frac{2}{256}$

8. [21] (**True/False.**) For each of the following statements, write "T" if it is true and "F" if it is false. (You need not give counterexamples for false statements.)

(a) _____ For all nonzero rational numbers a, b and all integers m, n:

$$a^m \cdot b^m = (ab)^{2m}.$$

(b) _____ For all integers a, b, and d: If $d|a$ and $d|b$, then $d|(a - b)$.

(c) _____ For all integers a and b: If p is a prime and $p|ab$ then $p|a$ or $p|b$.

(d) _____ For all integers a, b, and d: If $d|ab$, then $d|a$ or $d|b$.

(e) _____ For all integers n: If $3|n$ and $7|n$, then $21|n$.

(f) _____ For all integers n: If $4|n$ and $6|n$, then $24|n$.

(g) _____ For all integers a, b: If neither a nor b is even, then $\text{GCD}(a, b) \neq 2$.